chapter two

TRANSFORMERS

A transformer is a static machine. Although it is not an energy conversion device, it is indispensable in many energy conversion systems. It is a simple device, having two or more electric circuits coupled by a common magnetic circuit. Analysis of transformers involves many principles that are basic to the understanding of electric machines. Transformers are so widely used as electrical apparatus that they are treated along with other electric machines in most books on electric machines.

A transformer essentially consists of two or more windings coupled by a mutual magnetic field. Ferromagnetic cores are used to provide tight magnetic coupling and high flux densities. Such transformers are known as *iron core* transformers. They are invariably used in high-power applications. *Air core* transformers have poor magnetic coupling and are sometimes used in low-power electronic circuits. In this chapter we primarily discuss iron core transformers.

Two types of core constructions are normally used, as shown in Fig. 2.1. In the *core type* (Fig. 2.1*a*), the windings are wound around two legs of a magnetic core of rectangular shape. In the *shell type* (Fig. 2.1*b*), the windings are wound around the center leg of a three-legged magnetic core. To reduce core losses, the magnetic core is formed of a stack of thin laminations. Silicon-steel laminations of 0.014-inch thickness are commonly used for transformers operating at frequencies below a few hundred cycles. L-shaped laminations are used for core-type construction and E-shaped laminations are used for shell-type construction. To avoid a continuous air gap (which would require a large exciting current), laminations are stacked alternately, as shown in Figs. 2.1*c* and 2.1*d*.

For small transformers used in communication circuits at high frequencies (kilocycles to megacycles) and low power levels, compressed powdered ferromagnetic alloys, known as permalloy, are used.

A schematic representation of a two-winding transformer is shown in Fig. 2.2. The two vertical bars are used to signify tight magnetic coupling between the windings. One winding is connected to an ac supply and is referred to as the *primary winding*. The other winding is connected to an electrical load and is referred to as the *secondary winding*. The winding with the higher number of turns will have a high voltage and is called the high-voltage (HV) or high-tension (HT) winding. The winding with the lower number of turns is called the low-voltage (LV) or low-tension (LT) winding. To achieve tighter magnetic coupling between the windings, they may be formed of coils placed one on top of another (Fig. 2.1*a*) or side by side (Fig. 2.1*b*) in a "pancake" coil formation where primary and secondary coils are interleaved. Where the coils are placed one on top of another, the low-voltage winding is placed nearer the core and the high-voltage winding on top.

Transformers have widespread use. Their primary function is to change voltage level. Electrical power is generated in a power house at about 30, 000 volts. However, in domestic houses, electric power is used at 110 or 220 volts. Electric power is transmitted from a power plant to a load center at 200,000 to 500,000 volts. Transformers are used to step up and step down voltage

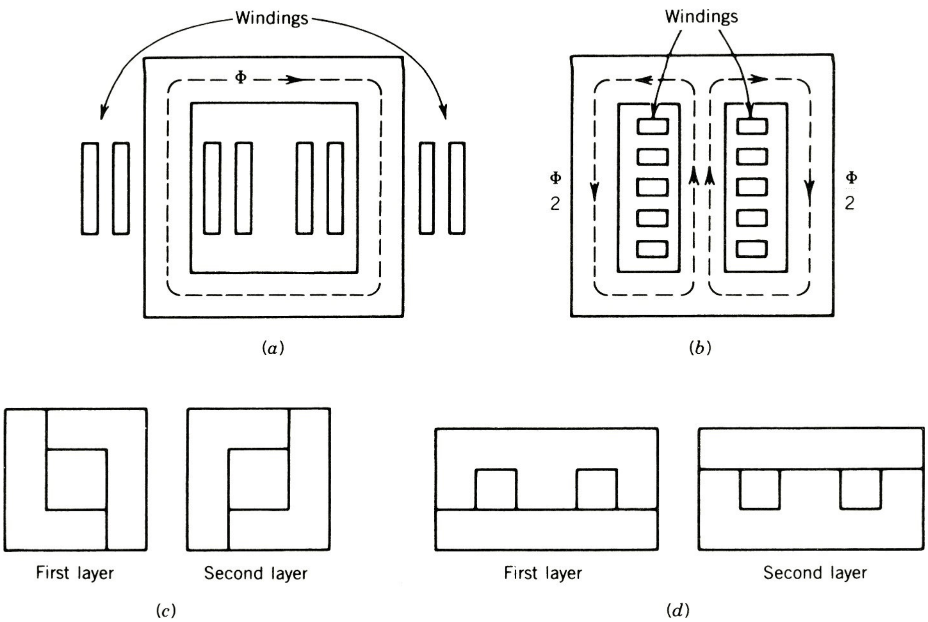


FIGURE 2.1 Transformer core construction. (*a*) Core-type, (*b*) Shell-type, (*c*) L-shaped lamination, (*d*) E-shaped lamination.

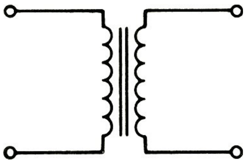


FIGURE 2.2 Schematic representation of a two-winding transformer.

at various stages of power transmission, as shown in Fig. 2.3. A large power transformer used to step up generator voltage from 24 to 345 kV is shown in Fig. 2.4. A distribution transformer used in a public utility system to step down voltage from 4.6 kV to 120 V is shown in Fig. 2.5.

Transformers are widely used in low-power electronic or control circuits to isolate one circuit from another circuit or to match the impedance of a source with its load for maximum power transfer. Transformers are also used to measure voltages and currents; these are known as *instrument transformers*.

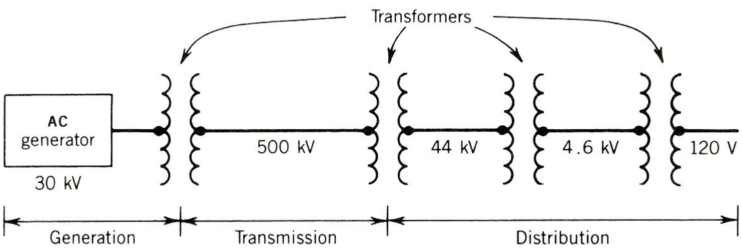


FIGURE 2.3 Power transmission using transformers.

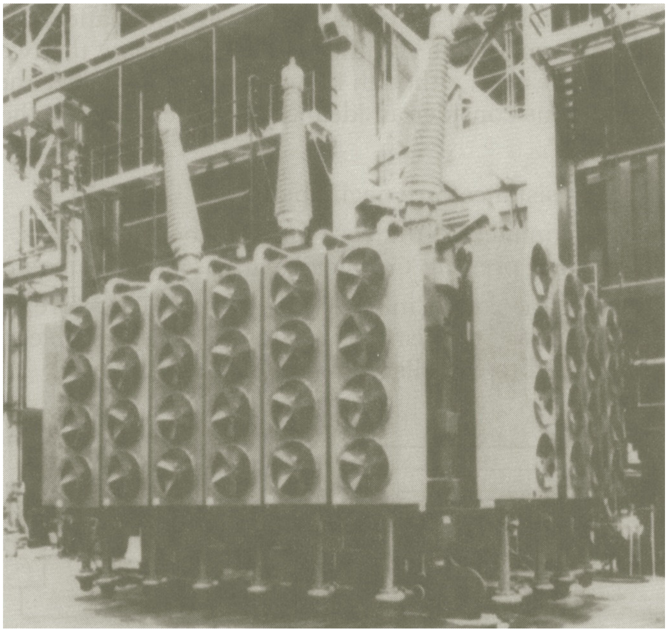


FIGURE 2.4 Power transformer, 24 to 345 kV.

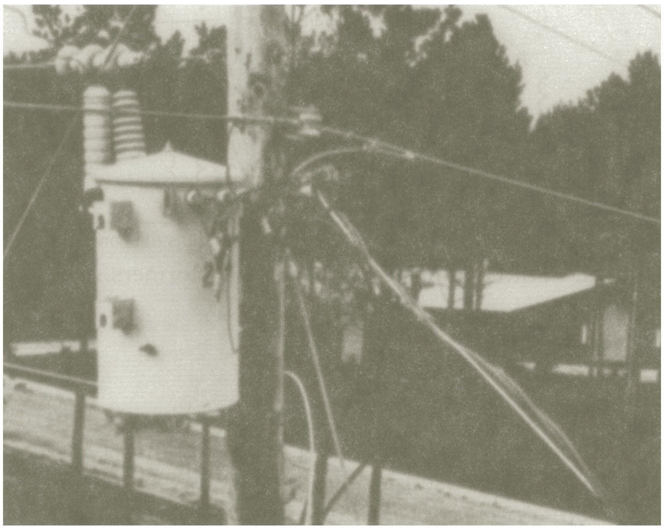


FIGURE 2.5 Distribution transformer.

2.1 IDEAL TRANSFORMER

Consider a transformer with two windings, a primary winding of N_1 turns and a secondary winding of N_2 turns, as shown schematically in Fig. 2.6. In a schematic diagram it is a common practice to show the two windings in the two legs of the core, although in an actual transformer

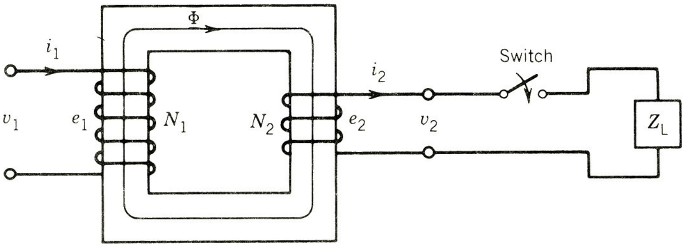


FIGURE 2.6 Ideal transformer.

the windings are interleaved. Let us consider an ideal transformer that has the following properties:

1. The winding resistances are negligible.
2. All fluxes are confined to the core and link both windings; that is, no leakage fluxes are present. Core losses are assumed to be negligible.
3. Permeability of the core is infinite (i.e., $\mu \rightarrow \infty$). Therefore, the exciting current required to establish flux in the core is negligible; that is, the net mmf required to establish a flux in the core is zero.

When the primary winding is connected to a time-varying voltage v_1, a time-varying flux Φ is established in the core. A voltage e_1 will be induced in the winding and will equal the applied voltage if resistance of the winding is neglected:

$$v_1 = e_1 = N_1 \frac{d\Phi}{dt} \tag{2.1}$$

The core flux also links the secondary winding and induces a voltage e_2, which is the same as the terminal voltage v_2:

$$v_2 = e_2 = N_2 \frac{d\Phi}{dt} \tag{2.2}$$

From Eqs. 2.1 and 2.2,

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = a \tag{2.3}$$

where a is the turns ratio.

Equation 2.3 indicates that the voltages in the windings of an ideal transformer are directly proportional to the turns of the windings.

Let us now connect a load (by closing the switch in Fig. 2.6) to the secondary winding. A current i_2 will flow in the secondary winding, and the secondary winding will provide an mmf $N_2 i_2$ for the core. This will immediately make a primary winding current i_1 flow so that a counter-mmf $N_1 i_1$ can oppose $N_2 i_2$. Otherwise $N_2 i_2$ would make the core flux change drastically and the balance between v_1 and e_1 would be disturbed. Note in Fig. 2.6 that the current directions are shown such that their mmfs oppose each other. Because the net mmf required to establish a flux in the ideal core is zero,

$$N_1 i_1 - N_2 i_2 = \text{net mmf} = 0 \tag{2.4}$$

$$N_1 i_1 = N_2 i_2 \tag{2.5}$$

$$\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{2.6}$$

The currents in the windings are inversely proportional to the turns of the windings. Also note that if more current is drawn by the load, more current will flow from the supply. It is this mmf-balancing requirement (Eq. 2.5) that makes the primary know of the presence of current in the secondary.

From Eqs. 2.3 and 2.6,

$$v_1 i_1 = v_2 i_2 \tag{2.7}$$

That is, the instantaneous power input to the transformer equals the instantaneous power output from the transformer. This is expected, because all power losses are neglected in an ideal transformer. Note that although there is no physical connection between load and supply, as soon as power is consumed by the load, the same power is drawn from the supply. The transformer, therefore, provides a physical isolation between load and supply while maintaining electrical continuity.

If the supply voltage v_1 is sinusoidal, then Eqs. 2.3, 2.6, and 2.7 can be written in terms of rms values:

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = a \tag{2.8}$$

$$\frac{I_1}{I_2} = \frac{N_2}{N_1} = \frac{1}{a} \tag{2.9}$$

$$\underset{\substack{\uparrow \\ \text{input} \\ \text{volt-amperes}}}{V_1 I_1} = \underset{\substack{\uparrow \\ \text{output} \\ \text{volt-amperes}}}{V_2 I_2} \tag{2.10}$$

2.1.1 IMPEDANCE TRANSFER

Consider the case of a sinusoidal applied voltage and a secondary impedance Z_2, as shown in Fig. 2.7*a*.

$$Z_2 = \frac{V_2}{I_2}$$

The input impedance is

$$Z_1 = \frac{V_1}{I_1} = \frac{aV_2}{I_2/a} = a^2 \frac{V_2}{I_2}$$

$$= a^2 Z_2 \tag{2.11}$$

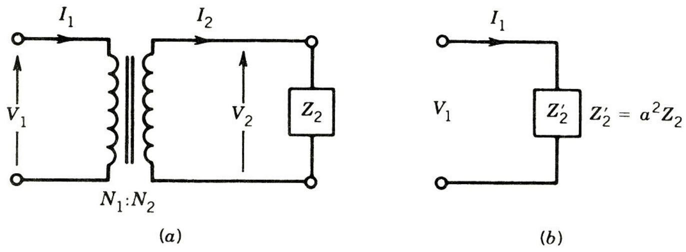


FIGURE 2.7 Impedance transfer across an ideal transformer.

so

$$Z_1 = a^2 Z_2 = Z_2' \tag{2.12}$$

An impedance Z_2 connected in the secondary will appear as an impedance Z_2' looking from the primary. The circuit in Fig. 2.7*a* is therefore equivalent to the circuit in Fig. 2.7*b*. Impedance can be transferred from secondary to primary if its value is multiplied by the square of the turns ratio. An impedance from the primary side can also be transferred to the secondary side, and in that case its value has to be divided by the square of the turns ratio:

$$Z_1' = \frac{1}{a^2} Z_1 \tag{2.13}$$

This impedance transfer is very useful because it eliminates a coupled circuit in an electrical circuit and thereby simplifies the circuit.

EXAMPLE 2.1

A speaker of 9 Ω, resistive impedance is connected to a supply of 10 V with internal resistive impedance of 1 Ω, as shown in Fig. E2.1*a*.

(a) Determine the power absorbed by the speaker.

(b) To maximize the power transfer to the speaker, a transformer of 1 : 3 turns ratio is used between source and speaker as shown in Fig. E2.1*b*. Determine the power taken by the speaker.

Solution

(a) From Fig. E2.1*a*,

$$I = \frac{10}{1+9} = 1 \text{ A}$$

$$P = I^2 \times 9 = 9 \text{ W}$$

(b) If the resistance of the speaker is referred to the primary side, its resistance is

$$R_2' = a^2 R_1 = \left(\frac{1}{3}\right)^2 \times 9 = 1\ \Omega$$

The equivalent circuit is shown in Fig. E2.1*c*.

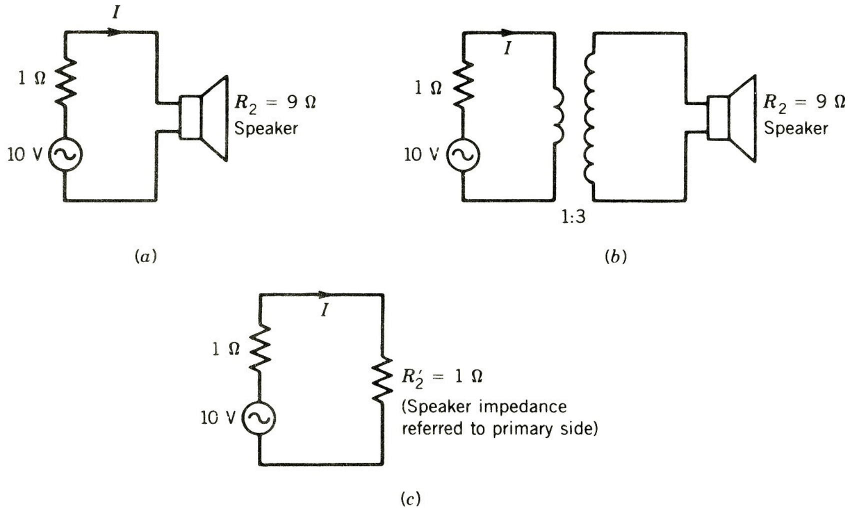


FIGURE E2.1

$$I = \frac{10}{1+1} = 5 \text{ A}$$

$$P = 5^2 \times 1 = 25 \text{ W} \quad \blacksquare$$

2.1.2 POLARITY

Windings on transformers or other electrical machines are marked to indicate terminals of like polarity. Consider the two windings shown in Fig. 2.8*a*. Terminals 1 and 3 are identical, because currents entering these terminals produce fluxes in the same direction in the core that forms the common magnetic path. For the same reason, terminals 2 and 4 are identical. If these two windings are linked by a common time-varying flux, voltages will be induced in these windings such that if at a particular instant the potential of terminal 1 is positive with respect to terminal 2, then at the same instant the potential of terminal 3 will be positive with respect to terminal 4. In other words, induced voltages e_{12} and e_{34} are in phase. Identical terminals such as 1 and 3 or 2 and 4 are sometimes marked by dots or ± as shown in Fig. 2.8*b*. These are called the polarity markings of the windings. They indicate how the windings are wound on the core.

If the windings can be visually seen in a machine, the polarities can be determined. However, usually only the terminals of the windings are brought outside the machine. Nevertheless, it is possible to determine the polarities of the windings experimentally. A simple method is illustrated in Fig. 2.8*c*, in which terminals 2 and 4 are connected together and winding 1–2 is connected to an ac supply.

The voltages across 1–2, 3–4, and 1–3 are measured by a voltmeter. Let these voltage readings be called V_{12},V_{34}, and V_{13}, respectively. If a voltmeter reading V_{13} is the sum of voltmeter readings V_{12} and V_{34} (i.e., $V_{13} \simeq V_{12} + V_{34}$), it means that at any instant when the potential of terminal 1 is positive with respect to terminal 2, the potential of terminal 4 is positive with respect to terminal 3. The induced voltages e_{12} and e_{43} are in phase, as shown in Fig. 2.8*c*, making $e_{13} = e_{12} + e_{43}$. Consequently, terminals 1 and 4 are identical (or same polarity)

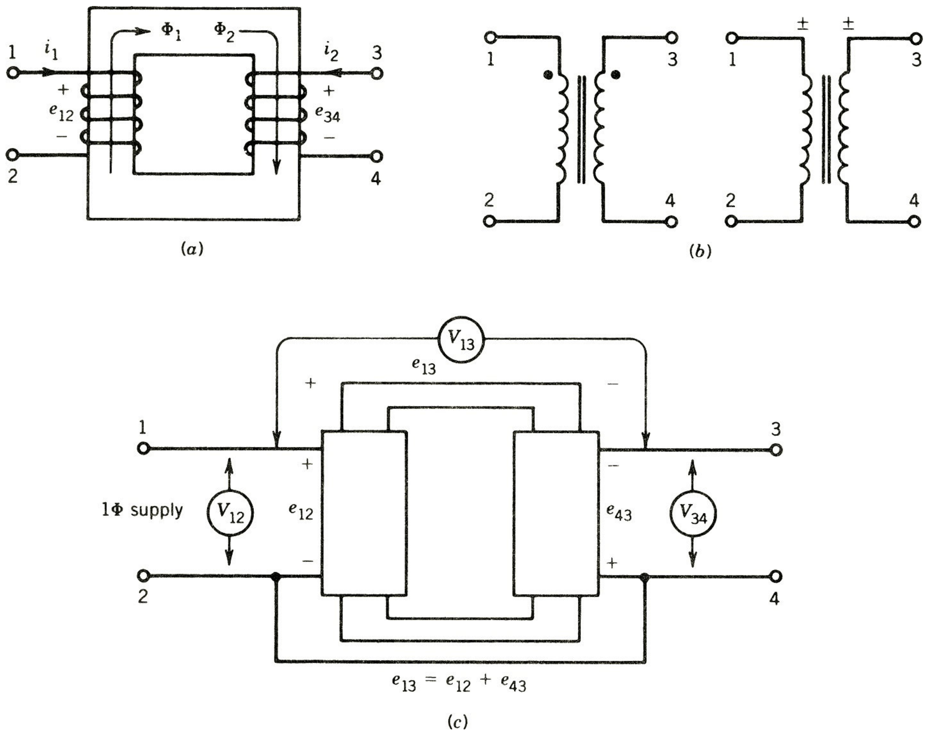


FIGURE 2.8 Polarity determination.

terminals. If the voltmeter reading V_{13} is the difference between voltmeter readings V_{12} and V_{34} (i.e., $V_{13} \simeq V_{12} - V_{34}$), then 1 and 3 are terminals of the same polarity.

Polarities of windings must be known if transformers are connected in parallel to share a common load. Figure 2.9*a* shows the parallel connection of two single-phase (1ϕ) transformers. This is the correct connection because secondary voltages e_{21} and e_{22} oppose each other internally. The connection shown in Fig. 2.9*b* is wrong, because e_{21} and e_{22} aid each

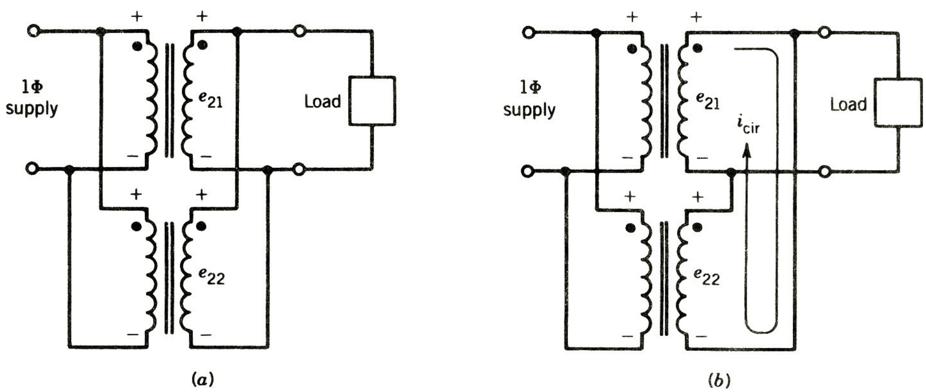


FIGURE 2.9 Parallel operation of single-phase transformers. (*a*) Correct connection. (*b*) Wrong connection.

other internally and a large circulating current i_{cir} will flow in the windings and may damage the transformers. For three-phase connection of transformers (see Section 2.6), the winding polarities must also be known.

2.2 PRACTICAL TRANSFORMER

In Section 2.1, the properties of an ideal transformer were discussed. Certain assumptions were made that are not valid in a practical transformer. For example, in a practical transformer the windings have resistances, not all windings link the same flux, permeability of the core material is not infinite, and core losses occur when the core material is subjected to time-varying flux. In the analysis of a practical transformer, all these imperfections must be considered.

Two methods of analysis can be used to account for the departures from the ideal transformer:

1. An equivalent circuit model based on physical reasoning.
2. A mathematical model based on the classical theory of magnetically coupled circuits.

Both methods will provide the same performance characteristics for the practical transformer. However, the equivalent circuit approach provides a better appreciation and understanding of the physical phenomena involved, and this technique will be presented here.

A practical winding has a resistance, and this resistance can be shown as a lumped quantity in series with the winding (Fig. 2.10*a*). When currents flow through windings in the transformer, they establish a resultant mutual (or common) flux Φ_m that is confined essentially to

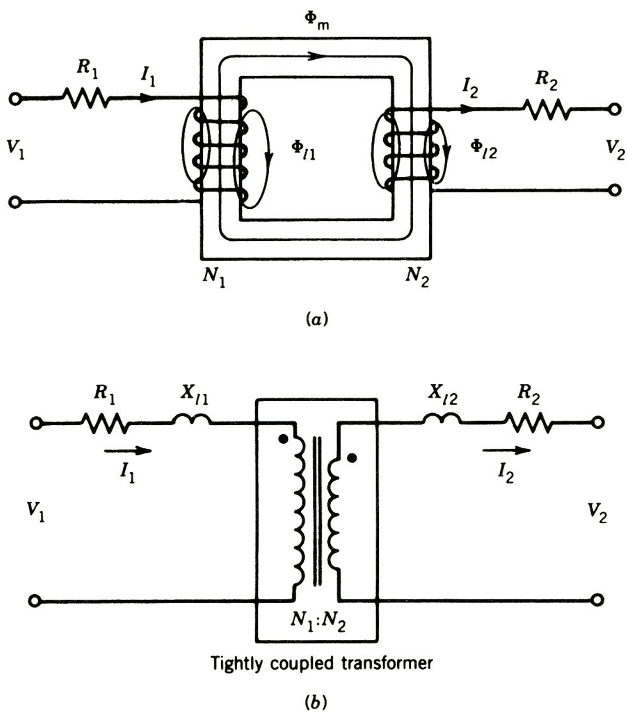


FIGURE 2.10 Development of the transformer equivalent circuits.

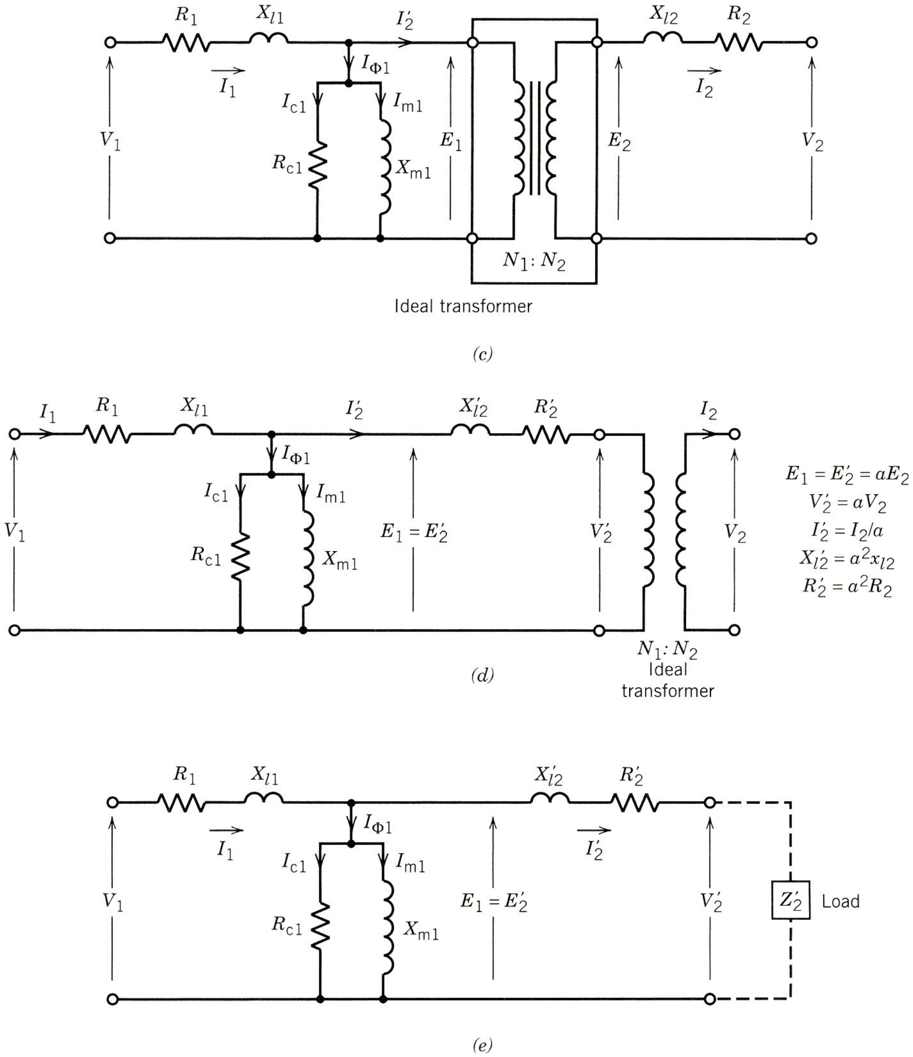


FIGURE 2.10 (*Continued*)

the magnetic core. However, a small amount of flux known as leakage flux, Φ_l (shown in Fig. 2.10*a*), links only one winding and does not link the other winding. The leakage path is primarily in air, and therefore the leakage flux varies linearly with current. The effects of leakage flux can be accounted for by an inductance, called leakage inductance:

$$L_{l1} = \frac{N_1 \Phi_{l1}}{i_1} = \text{leakage inductance of winding 1}$$

$$L_{l2} = \frac{N_2 \Phi_{l2}}{i_2} = \text{leakage inductance of winding 2}$$

If the effects of winding resistance and leakage flux are respectively accounted for by resistance R and leakage reactance $X_l(=2\pi f L_l)$, as shown in Fig. 2.10*b*, the transformer windings are tightly coupled by a mutual flux.

In a practical magnetic core having finite permeability, a magnetizing current I_{m} is required to establish a flux in the core. This effect can be represented by a magnetizing inductance L_{m}. Also, the core loss in the magnetic material can be represented by a resistance R_{c}. If these imperfections are also accounted for, then what we are left with is an ideal transformer, as shown in Fig. 2.10*c*. A practical transformer is therefore equivalent to an ideal transformer plus external impedances that represent imperfections of an actual transformer.

2.2.1 REFERRED EQUIVALENT CIRCUITS

The ideal transformer in Fig. 2.10*c* can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. This is almost invariably done. The equivalent circuit with the ideal transformer moved to the right is shown in Fig. 2.10*d*. For convenience, the ideal transformer is usually not shown and the equivalent circuit is drawn, as shown in Fig. 2.10*e*, with all quantities (voltages, currents, and impedances) referred to one side. The referred quantities are indicated with primes. By analyzing this equivalent circuit the referred quantities can be evaluated, and the actual quantities can be determined from them if the turns ratio is known.

Approximate Equivalent Circuits

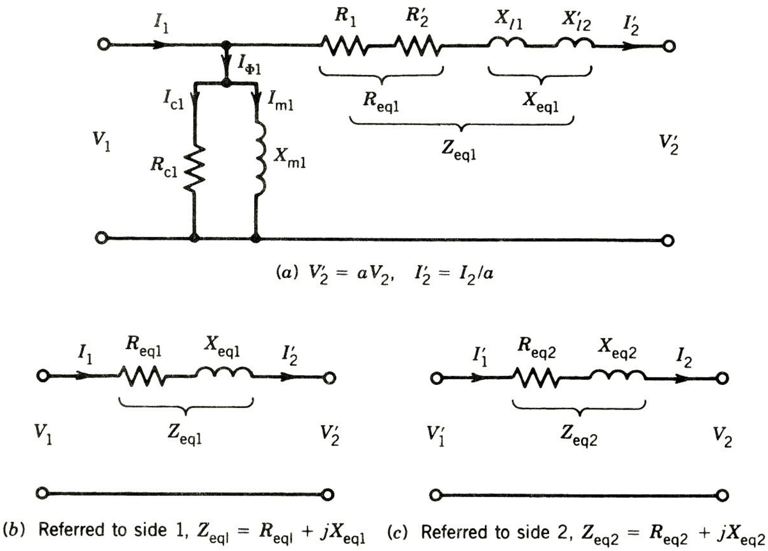
The voltage drops I_1R_1 and I_1X_{l1} (Fig. 2.10*e*) are normally small and $|E_1| \simeq |V_1|$. If this is true, then the shunt branch (composed of R_{c1} and X_{m1}) can be moved to the supply terminal, as shown in Fig. 2.11*a*. This approximate equivalent circuit simplifies computation of currents, because both the exciting branch impedance and the load branch impedance are directly connected across the supply voltage. Besides, the winding resistances and leakage reactances can be lumped together. This equivalent circuit (Fig. 2.11*a*) is frequently used to determine the performance characteristics of a practical transformer.

In a transformer, the exciting current I_ϕ is a small percentage of the rated current of the transformer (less than 5%). A further approximation of the equivalent circuit can be made by removing the excitation branch, as shown in Fig. 2.11*b*. The equivalent circuit referred to side 2 is also shown in Fig. 2.11*c*.

2.2.2 DETERMINATION OF EQUIVALENT CIRCUIT PARAMETERS

The equivalent circuit model (Fig. 2.10*e*) for the actual transformer can be used to predict the behavior of the transformer. The parameters $R_1, X_{l1}, R_{\mathrm{c1}}, X_{\mathrm{m1}}, R_2, X_{l2}$, and $a\ (=N_1/N_2)$ must be known so that the equivalent circuit model can be used.

If the complete design data of a transformer are available, these parameters can be calculated from the dimensions and properties of the materials used. For example, the winding resistances (R_1, R_2) can be calculated from the resistivity of copper wires, the total length, and the cross-sectional area of the winding. The magnetizing inductances L_{m} can be calculated from the number of turns of the winding and the reluctance of the magnetic path. The calculation of the leakage inductance (L_l) will involve accounting for partial flux linkages and is therefore



$$R_{eq2} = \frac{R_{eq1}}{a^2} = R_2 + R_1'$$

$$X_{eq2} = \frac{X_{eq1}}{a^2} = X_{l2} + X_{l1}'$$

$$V_1' = \frac{V_1}{a}, \quad I_1' = I_2 = aI_1$$

FIGURE 2.11 Approximate equivalent circuits.

complicated. However, formulas are available from which a reliable determination of these quantities can be made.

These parameters can be directly and more easily determined by performing tests that involve little power consumption. Two tests, a no-load test (or open-circuit test) and a short-circuit test, will provide information for determining the parameters of the equivalent circuit of a transformer, as will be illustrated by an example.

Transformer Rating

The kilovolt–ampere (kVA) rating and voltage ratings of a transformer are marked on its nameplate. For example, a typical transformer may carry the following information on the nameplate: 10 kVA, 1100/110 volts. What are the meanings of these ratings? The voltage ratings indicate that the transformer has two windings, one rated for 1100 volts and the other for 110 volts. These voltages are proportional to their respective numbers of turns, and therefore the voltage ratio also represents the turns ratio ($a = 1100/110 = 10$). The 10 kVA rating means that each winding is designed for 10 kVA. Therefore, the current rating for the high-voltage winding is $10,000/1100 = 9.09$ A and for the lower-voltage winding is $10,000/110 = 90.9$ A. It may be noted that when the rated current of 90.9 A flows through the low-voltage winding, the rated current of 9.09 A will flow through the high-voltage winding. In an actual case, however, the winding that is connected to the supply (called the primary winding) will carry an additional component of current (excitation current), which is very small compared to the rated current of the winding.

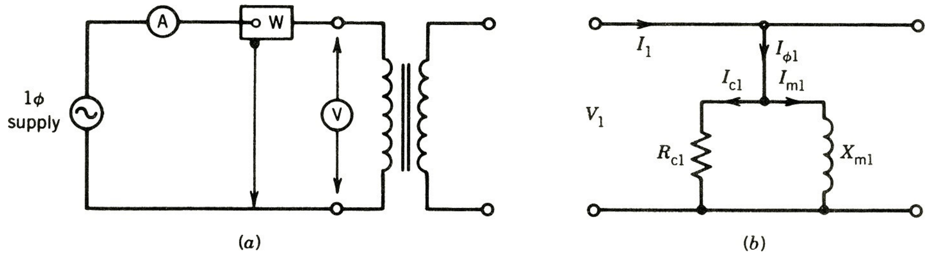


FIGURE 2.12 No-load (or open-circuit) test. (*a*) Wiring diagram for open-circuit test. (*b*) Equivalent circuit under open circuit.

No-Load Test (or Open-Circuit Test)

This test is performed by applying a voltage to either the high-voltage side or low-voltage side, whichever is convenient. Thus, if a 1100/110 volts transformer were to be tested, the voltage would be applied to the low-voltage winding, because a power supply of 110 volts is more readily available than a supply of 1100 volts.

A wiring diagram for open-circuit test of a transformer is shown in Fig. 2.12*a*. Note that the secondary winding is kept open. Therefore, from the transformer equivalent circuit of Fig. 2.11*a* the equivalent circuit under open-circuit conditions is as shown in Fig. 2.12*b*. The primary current is the exciting current and the losses measured by the wattmeter are essentially the core losses. The equivalent circuit of Fig. 2.12*b* shows that the parameters R_c and X_m can be determined from the voltmeter, ammeter, and wattmeter readings.

Note that the core losses will be the same whether 110 volts are applied to the low-voltage winding having the smaller number of turns or 1100 volts are applied to the high-voltage winding having the larger number of turns. The core loss depends on the maximum value of flux in the core, which is the same in either case, as indicated by Eq. 1.40.

Short-Circuit Test

This test is performed by short-circuiting one winding and applying rated current to the other winding, as shown in Fig. 2.13*a*. In the equivalent circuit of Fig. 2.11*a* for the transformer, the impedance of the excitation branch (shunt branch composed of R_c and X_m) is much larger than that of the series branch (composed of R_{eq} and X_{eq}). If the secondary terminals are shorted, the high impedance of the shunt branch can be neglected. The equivalent circuit with the secondary short-circuited can thus be represented by the circuit shown in Fig. 2.13*b*. Note

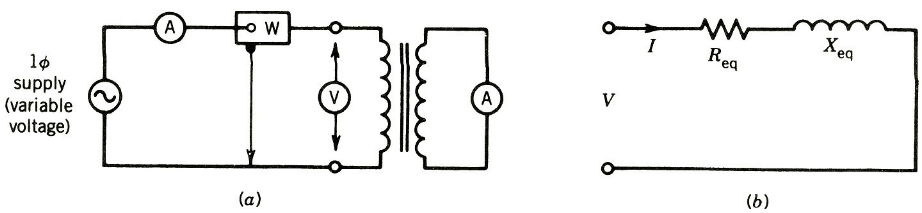


FIGURE 2.13 Short-circuit test. (*a*) Wiring diagram for short-circuit test. (*b*) Equivalent circuit at short-circuit condition.

that since $Z_{eq} (= R_{eq} + jX_{eq})$ is small, only a small supply voltage is required to pass rated current through the windings. It is convenient to perform this test by applying a voltage to the high-voltage winding.

As can be seen from Fig. 2.13*b*, the parameters R_{eq} and X_{eq} can be determined from the readings of voltmeter, ammeter, and wattmeter. In a well-designed transformer, $R_1 = a^2R_1 = R_2'$ and $X_{l1} = a^2X_{l2} = X_{l2}'$. Note that because the voltage applied under the short-circuit condition is small, the core losses are neglected, and the wattmeter reading can be taken entirely to represent the copper losses in the windings, represented by R_{eq}.

The following example illustrates the computation of the parameters of the equivalent circuit of a transformer.

EXAMPLE 2.2

Tests are performed on a 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer and the following results are obtained.

	Open-Circuit Test (high-voltage side open)	Short-Circuit Test (low-voltage side shorted)
Voltmeter	220 V	150 V
Ammeter	2.5 A	4.55 A
Wattmeter	100 W	215 W

(a) Derive the parameters for the approximate equivalent circuits referred to the low-voltage side and the high-voltage side.

(b) Express the excitation current as a percentage of the rated current.

(c) Determine the power factor for the no-load and short-circuit tests.

Solution

Note that for the no-load test the supply voltage (full-rated voltage of 220 V) is applied to the low-voltage winding, and for the short-circuit test the supply voltage is applied to the high-voltage winding with the low-voltage winding shorted. The subscripts H and L will be used to represent quantities for the high-voltage and low-voltage windings, respectively.

The ratings of the windings are as follows:

$$V_{\mathrm{H(rated)}} = 2200 \text{ V}$$

$$V_{\mathrm{L(rated)}} = 220 \text{ V}$$

$$I_{\mathrm{H(rated)}} = \frac{10{,}000}{2200} = 4.55 \text{ A}$$

$$I_{\mathrm{L(rated)}} = \frac{10{,}000}{220} = 45.5 \text{ A}$$

$$\mathrm{V_H} I_{\mathrm{H}}|_{\mathrm{rated}} = V_{\mathrm{L}} I_{\mathrm{L}}|_{\mathrm{rated}} = 10 \text{ kVA}$$

(a) The equivalent circuit and the phasor diagram for the open-circuit test are shown in Fig. E2.2*a*.

$$\text{Power, } P_{\text{oc}} = \frac{V_{\text{L}}^2}{R_{\text{cL}}}$$

$$R_{\text{cL}} = \frac{220^2}{100} = 484\ \Omega$$

$$I_{\text{cL}} = \frac{220}{484} = 0.45\ \text{A}$$

$$I_{\text{mL}} = (I_{\text{L}}^2 - I_{\text{cL}}^2)^{1/2} = (2.5^2 - 0.45^2)^{1/2} = 2.46\ \text{A}$$

$$X_{\text{mL}} = \frac{V_{\text{L}}}{I_{\text{mL}}} = \frac{220}{2.46} = 89.4\ \Omega$$

The corresponding parameters for the high-voltage side are obtained as follows:

$$\text{Turns ratio } a = \frac{2200}{220} = 10$$

$$R_{\text{cH}} = a^2 R_{\text{cL}} = 10^2 \times 484 = 48{,}400\ \Omega$$

$$X_{\text{mH}} = 10^2 \times 89.4 = 8940\ \Omega$$

The equivalent circuit with the low-voltage winding shorted is shown in Fig. E2.2*b*.

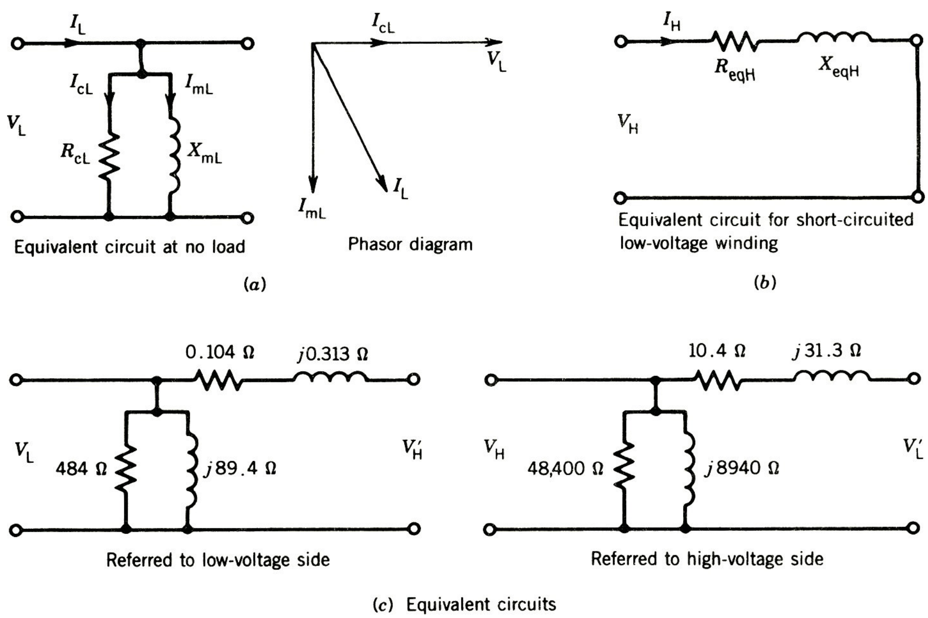


FIGURE E2.2

$$\text{Power } P_{sc} = I_H^2 R_{eqH}$$

$$R_{eqH} = \frac{215}{4.55^2} = 10.4\ \Omega$$

$$Z_{eqH} = \frac{V_H}{I_H} = \frac{150}{4.55} = 32.97\ \Omega$$

$$X_{eqH} = (Z_{eqH}^2 - R_{eqH}^2)^{1/2} = (32.97^2 - 10.4^2)^{1/2} = 31.3\ \Omega$$

The corresponding parameters for the low-voltage side are as follows:

$$R_{eqL} = \frac{R_{eqH}}{a^2} = \frac{10.4}{10^2} = 0.104\ \Omega$$

$$X_{eqL} = \frac{31.3}{10^2} = 0.313\ \Omega$$

The approximate equivalent circuits referred to the low-voltage side and the high-voltage side are shown in Fig. E2.2*c*. Note that the impedance of the shunt branch is much larger than that of the series branch.

(b) From the no-load test the excitation current, with rated voltage applied to the low-voltage winding, is

$$I_\phi = 2.5\ \text{A}$$

This is $(2.5/45.5) \times 100\% = 5.5\%$ of the rated current of the winding.

(c)

$$\begin{aligned}\text{Power factor at no load} &= \frac{\text{power}}{\text{volt-ampere}}\\ &= \frac{100}{220 \times 2.5}\\ &= 0.182\end{aligned}$$

$$\text{Power factor at short-circuit condition} = \frac{215}{150 \times 4.55} = 0.315 \quad \blacksquare$$

2.3 VOLTAGE REGULATION

Most loads connected to the secondary of a transformer are designed to operate at essentially constant voltage. However, as the current is drawn through the transformer, the load terminal voltage changes because of voltage drop in the internal impedance of the transformer. Consider Fig. 2.14*a*, where the transformer is represented by a series impedance Z_{eq}. If a load

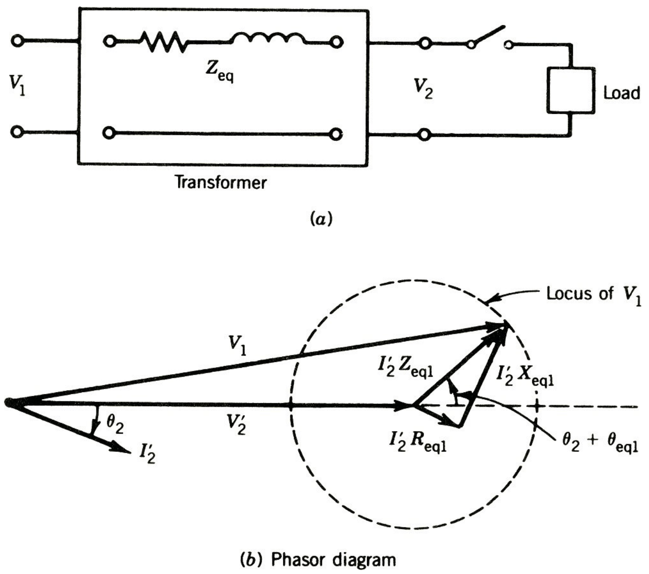


FIGURE 2.14 Voltage regulation.

is not applied to the transformer (i.e., open-circuit or no-load condition), the load terminal voltage is

$$V_2|_{NL} = \frac{V_1}{a} \tag{2.14}$$

If the load switch is now closed and the load is connected to the transformer secondary, the load terminal voltage is

$$V_2|_L = V_2|_{NL} \pm \Delta V_2 \tag{2.15}$$

The load terminal voltage may go up or down depending on the nature of the load. This voltage change is due to the voltage drop (IZ) in the internal impedance of the transformer. A large voltage change is undesirable for many loads. For example, as more and more light bulbs are connected to the transformer secondary and the voltage decreases appreciably, the bulbs will glow with diminished illumination. To reduce the magnitude of the voltage change, the transformer should be designed for a low value of the internal impedance Z_{eq}.

A figure of merit called *voltage regulation* is used to identify this characteristic of voltage change in a transformer with loading. The voltage regulation is defined as the change in magnitude of the secondary voltage as the load current changes from the no-load to the loaded condition. This is expressed as follows:

$$\text{Voltage regulation} = \frac{|V_2|_{NL} - |V_2|_L}{|V_2|_L} \tag{2.16}$$

The absolute signs are used to indicate that it is the change in magnitudes that is important for the performance of the load. The voltages in Eq. 2.16 can be calculated by using equivalent circuits referred to either primary or secondary. Let us consider the equivalent circuit referred to the primary, shown in Fig. 2.11*b*. Equation 2.16 can also be written as

$$\text{Voltage regulation} = \frac{|V_2'|_{\mathrm{NL}} - |V_2'|_{\mathrm{L}}}{|V_2'|_{\mathrm{L}}} \tag{2.17}$$

The load voltage is normally taken as the rated voltage. Therefore,

$$|V_2'|_{\mathrm{L}} = |V_2'|_{\mathrm{rated}} \tag{2.18}$$

From Fig. 2.11*b*,

$$V_1 = V_2' + I_2'R_{\mathrm{eq1}} + jI_2'X_{\mathrm{eq1}} \tag{2.19}$$

If the load is thrown off ($I_1 = I_2' = 0$), V_1 will appear as V_2'. Hence,

$$|V_2'|_{\mathrm{NL}} = |V_1| \tag{2.20}$$

From Eqs. 2.17, 2.18, and 2.20,

$$\begin{matrix}\text{Voltage regulation}\\ \text{(in percent)}\end{matrix} = \frac{|V_1| - |V_2'|_{\mathrm{rated}}}{|V_2'|_{\mathrm{rated}}} \times 100\% \tag{2.21}$$

The voltage regulation depends on the power factor of the load. This can be appreciated from the phasor diagram of the voltages. Based on Eq. 2.19 and Fig. 2.11*b*, the phasor diagram is drawn in Fig. 2.14*b*. The locus of V_1 is a circle of radius $|I_2'Z_{\mathrm{eq1}}|$. The magnitude of V_1 will be maximum if the phasor $I_2'Z_{\mathrm{eq1}}$ is in phase with V_2'. That is,

$$\theta_2 + \theta_{\mathrm{eq1}} = 0 \tag{2.22}$$

where θ_2 is the angle of the load impedance
θ_{eq1} is the angle of the transformer equivalent impedance, Z_{eq1}.

From Eq. 2.22,

$$\theta_2 = -\theta_{\mathrm{eq1}} \tag{2.23}$$

Therefore, the maximum voltage regulation occurs if the power factor angle of the load is the same as the transformer equivalent impedance angle and the load power factor is lagging.

EXAMPLE 2.3

Consider the transformer in Example 2.2. Determine the voltage regulation in percent for the following load conditions.

(a) 75% full load, 0.6 power factor lagging.
(b) 75% full load, 0.6 power factor leading.
(c) Draw the phasor diagram for conditions (a) and (b).

Solution

Consider the equivalent circuit referred to the high-voltage side, as shown in Fig. E2.3. The load voltage is assumed to be at the rated value. The condition 75% full load means that the load current is 75% of the rated current. Therefore,

$$I_H = I'_L = 0.75 \times 4.55 = 3.41 \text{ A}$$

$$\text{Power factor} \quad PF = \cos\theta_2 = 0.6$$

$$\theta_2 = \pm 53.13$$

(a) For a lagging power factor, $\theta_2 = -53.13^\circ$

$$
\begin{aligned}
I_H &= 3.41\underline{/-53.13^\circ} \text{ A} \\
V_H &= V'_L + I'_L Z_{eqH} \\
&= 2200\underline{/0^\circ} + 3.41\underline{/-53.13^\circ}(10.4 + j31.3) \\
&= 2200 + 35.46\underline{/-53.13^\circ} + 106.73\underline{/90^\circ - 53.13^\circ} \\
&= 2200 + 21.28 - j28.37 + 85.38 + j64.04 \\
&= 2306.66 + j35.67 \\
&= 2306.94\underline{/0.9^\circ} \text{ V}
\end{aligned}
$$

$$\text{Voltage regulation} = \frac{2306.94 - 2200}{2200} \times 100\%$$
$$= 4.86\%$$

The meaning of 4.86% voltage regulation is that if the load is thrown off, the load terminal voltage will rise from 220 to 230.69 volts. In other words, when the 75% full load at

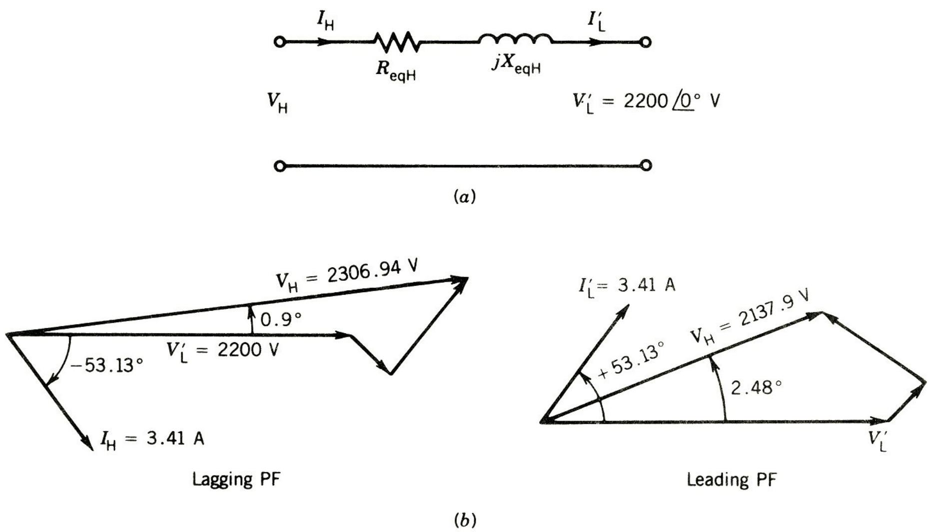


FIGURE E2.3

0.6 lagging power factor is connected to the load terminals of the transformer, the voltage drops from 230.69 to 220 volts.

(b) For leading power factor load, $\theta_2 = +53.13$

$$\begin{aligned} V_{\mathrm{H}} &= 2200\underline{/0^\circ} + 3.41\underline{/53.13^\circ}(10.4 + j31.3) \\ &= 2200 + 35.46\underline{/53.13^\circ} + 106.73\underline{/90^\circ + 53.13^\circ} \\ &= 2200 + 21.28 + j28.37 - 85.38 + j64.04 \\ &= 2135.9 + j92.41 \\ &= 2137.9\underline{/2.48^\circ}\ \mathrm{V} \end{aligned}$$

$$\begin{aligned} \text{Voltage regulation} &= \frac{2137.9 - 2200}{2200} \times 100\% \\ &= -2.82\% \end{aligned}$$

Note that the voltage regulation for this leading power factor load is negative. This means that if the load is thrown off, the load terminal voltage will decrease from 220 to 213.79 volts. To put it differently, if the leading power factor load is connected to the load terminals, the voltage will increase from 213.79 to 220 volts.

(c) The phasor diagrams for both lagging and leading power factor loads are shown in Fig. E2.3*b*. ■

2.4 EFFICIENCY

Equipment is desired to operate at a high efficiency. Fortunately, losses in transformers are small. Because the transformer is a static device, there are no rotational losses such as windage and friction losses in a rotating machine. In a well-designed transformer the efficiency can be as high as 99%. The efficiency is defined as follows:

$$\eta = \frac{\text{output power } (P_{\text{out}})}{\text{input power } (P_{\text{in}})} \tag{2.24}$$

$$= \frac{P_{\text{out}}}{P_{\text{out}} + \text{losses}} \tag{2.25}$$

The losses in the transformer are the core loss (P_{c}) and copper loss (P_{cu}). Therefore,

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_{\text{c}} + P_{\text{cu}}} \tag{2.26}$$

The copper loss can be determined if the winding currents and their resistances are known:

$$P_{\text{cu}} = I_1^2 R_1 + I_2^2 R_2 \tag{2.27}$$

$$= I_1^2 R_{\text{eq1}} \tag{2.27a}$$

$$= I_2^2 R_{\text{eq2}} \tag{2.27b}$$

The copper loss is a function of the load current.

The core loss depends on the peak flux density in the core, which in turn depends on the voltage applied to the transformer. Since a transformer remains connected to an essentially constant voltage, the core loss is almost constant and can be obtained from the no-load test of a transformer, as shown in Example 2.2. Therefore, if the parameters of the equivalent circuit of a transformer are known, the efficiency of the transformer under any operating condition may be determined. Now,

$$P_{\text{out}} = V_2 I_2 \cos\theta_2 \tag{2.28}$$

Therefore,

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + I_2^2 R_{\text{eq2}}} \tag{2.29}$$

Normally, load voltage remains fixed. Therefore, efficiency depends on load current (I_2) and load power factor ($\cos\theta_2$).

2.4.1 MAXIMUM EFFICIENCY

For constant values of the terminal voltage V_2 and load power factor angle θ_2, the maximum efficiency occurs when

$$\frac{d\eta}{dI_2} = 0 \tag{2.30}$$

If this condition is applied to Eq. 2.29, the condition for maximum efficiency is

$$P_c = I_2^2 R_{\text{eq2}} \tag{2.31}$$

That is, core loss = copper loss. For full-load condition,

$$P_{\text{cu,FL}} = I_{2,\text{FL}}^2 R_{\text{eq2}} \tag{2.31a}$$

Let

$$X = \frac{I_2}{I_{2,\text{FL}}} = \text{per unit loading} \tag{2.31b}$$

From Eqs. 2.31, 2.31a, and 2.31b,

$$P_c = X^2 P_{\text{cu,FL}}$$

$$X = \left(\frac{P_c}{P_{\text{cu,FL}}}\right)^{1/2} \tag{2.31c}$$

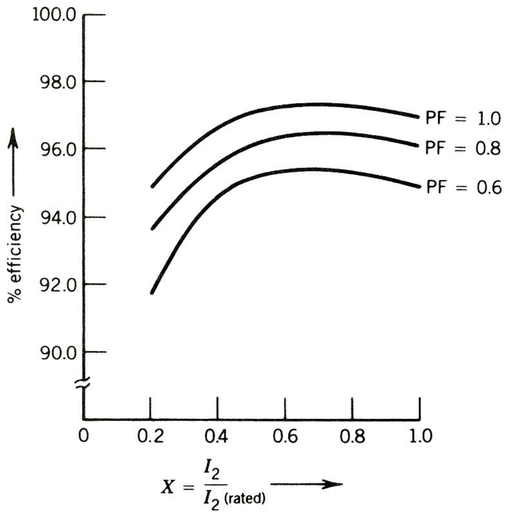


FIGURE 2.15 Efficiency of a transformer.

For constant values of the terminal voltage V_2 and load current I_2, the maximum efficiency occurs when

$$\frac{d\eta}{d\theta_2} = 0 \tag{2.32}$$

If this condition is applied to Eq. 2.29, the condition for maximum efficiency is

$$\theta_2 = 0 \tag{2.33}$$

$$\cos\theta_2 = 1 \tag{2.33a}$$

$$\text{that is, load power factor} = 1 \tag{2.33b}$$

Therefore, maximum efficiency in a transformer occurs when the load power factor is unity (i.e., resistive load) and load current is such that copper loss equals core loss. The variation of efficiency with load current and load power factor is shown in Fig. 2.15.

2.4.2 ALL-DAY (OR ENERGY) EFFICIENCY, η_{AD}

The transformer in a power plant usually operates near its full capacity and is taken out of circuit when it is not required. Such transformers are called *power transformers*, and they are usually designed for maximum efficiency occurring near the rated output. A transformer connected to the utility that supplies power to your house and the locality is called a *distribution transformer.* Such transformers are connected to the power system 24 hours a day and operate well below the rated power output for most of the time. It is therefore desirable to design a distribution transformer for maximum efficiency occurring at the average output power.

A figure of merit that will be more appropriate to represent the efficiency performance of a distribution transformer is the "all-day" or "energy" efficiency of the transformer. This is defined as follows:

$$\begin{aligned}\eta_{AD} &= \frac{\text{energy output over 24 hours}}{\text{energy input over 24 hours}} \\ &= \frac{\text{energy output over 24 hours}}{\text{energy output over 24 hours} + \text{losses over 24 hours}}\end{aligned} \tag{2.34}$$

If the load cycle of the transformer is known, the all-day efficiency can be determined.

EXAMPLE 2.4

For the transformer in Example 2.2, determine

(a) Efficiency at 75% rated output and 0.6 PF.

(b) Power output at maximum efficiency and the value of maximum efficiency. At what percent of full load does this maximum efficiency occur?

Solution

(a)

$$\begin{aligned}P_{out} &= V_2 I_2 \cos\theta_2 \\ &= 0.75 \times 10{,}000 \times 0.6 \\ &= 4500\ \text{W}\end{aligned}$$

$$P_c = 100\ \text{W (Example 2.2)}$$

$$\begin{aligned}P_{cu} &= I_H^2 R_{eqH} \\ &= (0.75 \times 4.55)^2 \times 10.4\ \text{W} \\ &= 121\ \text{W}\end{aligned}$$

$$\begin{aligned}\eta &= \frac{4500}{4500 + 100 + 121} \times 100\% \\ &= 95.32\%\end{aligned}$$

(b) At maximum efficiency,

$$P_{core} = P_{cu} \text{ and PF} = \cos\theta = 1$$

Now, $P_{core} = 100 \text{ W} = I_2^2 R_{eq2} = P_{cu}$.

$$I_2 = \left(\frac{100}{0.104}\right)^{1/2} = 31 \text{ A}$$

$$\begin{aligned} P_{out}\big|_{\eta_{max}} &= V_2 I_2 \cos\theta_2 \\ &= 220 \times 31 \times 1 \\ &= 6820 \text{ W} \end{aligned}$$

$$\eta_{max} = \frac{6820}{6820 + \underset{\substack{\uparrow \\ P_c}}{100} + \underset{\substack{\uparrow \\ P_{cu}}}{100}} \times 100\%$$

$$= 97.15\%$$

$$\text{Output kVA} = 6.82$$

$$\text{Rated kVA} = 10$$

η_{max} occurs at 68.2% full load.

Other Method

From Example 2.2,

$$P_{cu,FL} = 215 \text{ W}$$

From Eq. 2.31c,

$$X = \left(\frac{100}{215}\right)^{1/2} = 0.68 \quad \blacksquare$$

EXAMPLE 2.5

A 50 kVA, 2400/240 V transformer has a core loss $P_c = 200$ W at rated voltage and a copper loss $P_{cu} = 500$ W at full load. It has the following load cycle:

% Load	**0.0%**	**50%**	**75%**	**100%**	**110%**
Power factor		1	0.8 lag	0.9 lag	1
Hours	6	6	6	3	3

Determine the all-day efficiency of the transformer.

Solution

$$\begin{aligned}\text{Energy output over 24 hours} &= 0.5\times 50\times 6+0.75\times 50\times 0.8\\ &\quad \times 6+1\times 50\times 0.9\times 3+1.1\times 50\times 1\\ &\quad \times 3\text{ kWh}\\ &= 630\text{ kWh}\end{aligned}$$

Energy losses over 24 hours:

$$\begin{aligned}\text{Core loss} &= 0.2\times 24=4.8\text{ kWh}\\ \text{Copper loss} &= 0.5^2\times 0.5\times 6+0.75^2\times 0.5\times 6\\ &\quad +1^2\times 0.5\times 3+1.1^2\times 0.5\times 3\\ &= 5.76\text{ kWh}\\ \text{Total energy loss} &= 4.8+5.76=10.56\text{ kWh}\\ \eta_{\mathrm{AD}} &= \frac{630}{630+10.56}\times 100\%=98.35\%\end{aligned}$$ ■

2.5 AUTOTRANSFORMER

This is a special connection of the transformer from which a variable ac voltage can be obtained at the secondary. A common winding as shown in Fig. 2.16 is mounted on a core and the secondary is taken from a tap on the winding. In contrast to the two-winding transformer discussed earlier, the primary and secondary of an autotransformer are physically connected. However, the basic principle of operation is the same as that of the two-winding transformer.

Since all the turns link the same flux in the transformer core,

$$\frac{V_1}{V_2}=\frac{N_1}{N_2}=a \tag{2.35}$$

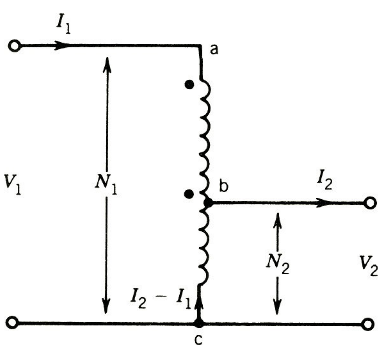


FIGURE 2.16 Autotransformer.

If the secondary tapping is replaced by a slider, the output voltage can be varied over the range $0 < V_2 < V_1$.

The ampere-turns provided by the upper half (i.e., by turns between points a and b) are

$$F_U = (N_1 - N_2)I_1 = \left(1 - \frac{1}{a}\right)N_1 I_1 \tag{2.36}$$

The ampere-turns provided by the lower half (i.e., by turns between points b and c) are

$$F_L = N_2(I_2 - I_1) = \frac{N_1}{a}(I_2 - I_1) \tag{2.37}$$

For ampere-turn balance, from Eqs. 2.36 and 2.37,

$$\left(1 - \frac{1}{a}\right)N_1 I_1 = \frac{N_1}{a}(I_2 - I_1)$$

$$\frac{I_1}{I_2} = \frac{1}{a} \tag{2.38}$$

Equations 2.35 and 2.38 indicate that, viewed from the terminals of the autotransformer, the voltages and currents are related by the same turns ratio as in a two-winding transformer.

The advantages of an autotransformer connection are lower leakage reactances, lower losses, lower exciting current, increased kVA rating (see Example 2.6), and variable output voltage when a sliding contact is used for the secondary. The disadvantage is the direct connection between the primary and secondary sides.

EXAMPLE 2.6

A 1ϕ, 100 kVA, 2000/200 V two-winding transformer is connected as an autotransformer as shown in Fig. E2.6 such that more than 2000 V is obtained at the secondary. The portion ab is the 200 V winding, and the portion bc is the 2000 V winding. Compute the kVA rating as an autotransformer.

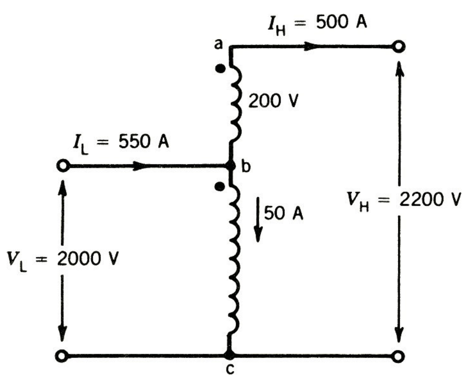


FIGURE E2.6

Solution

The current ratings of the windings are

$$I_{ab} = \frac{100{,}000}{200} \text{ A} = 500 \text{ A}$$

$$I_{bc} = \frac{100{,}000}{2000} = 50 \text{ A}$$

Therefore, for full-load operation of the autotransformer, the terminal currents are

$$I_H = 500 \text{ A}$$
$$I_L = 500 + 50 = 550 \text{ A}$$

Now, $V_L = 2000$ V and

$$V_H = 2000 + 200 = 2200 \text{ V}$$

Therefore,

$$\text{kVA}|_L = \frac{2000 \times 550}{1000} = 1100$$

$$\text{kVA}|_H = \frac{2200 \times 500}{1000} = 1100$$

A single-phase, 100 kVA, two-winding transformer when connected as an autotransformer can deliver 1100 kVA. Note that this higher rating of an autotransformer results from the conductive connection. Not all of the 1100 kVA is transformed by electromagnetic induction. Also note that the 200-volt winding must have sufficient insulation to withstand a voltage of 2200 V to ground. ■

2.6 THREE-PHASE TRANSFORMERS

A three-phase system is used to generate and transmit bulk electrical energy. Three-phase transformers are required to step up or step down voltages in the various stages of power transmission. A three-phase transformer can be built in one of two ways: by suitably connecting a bank of three single-phase transformers, or by constructing a three-phase transformer on a common magnetic structure.

2.6.1 BANK OF THREE SINGLE-PHASE TRANSFORMERS (THREE-PHASE TRANSFORMER BANK)

A set of three similar single-phase transformers may be connected to form a three-phase transformer. The primary and secondary windings may be connected in either wye (Y) or delta (Δ) configurations. There are therefore four possible connections for a three-phase transformer: Y–Δ, Δ–Y, Δ–Δ, and Y–Y. Figure 2.17*a* shows a Y–Δ connection of a three-phase transformer. On the primary side, three terminals of identical polarity are connected together to form the neutral of the Y connection. On the secondary side, the windings are connected in

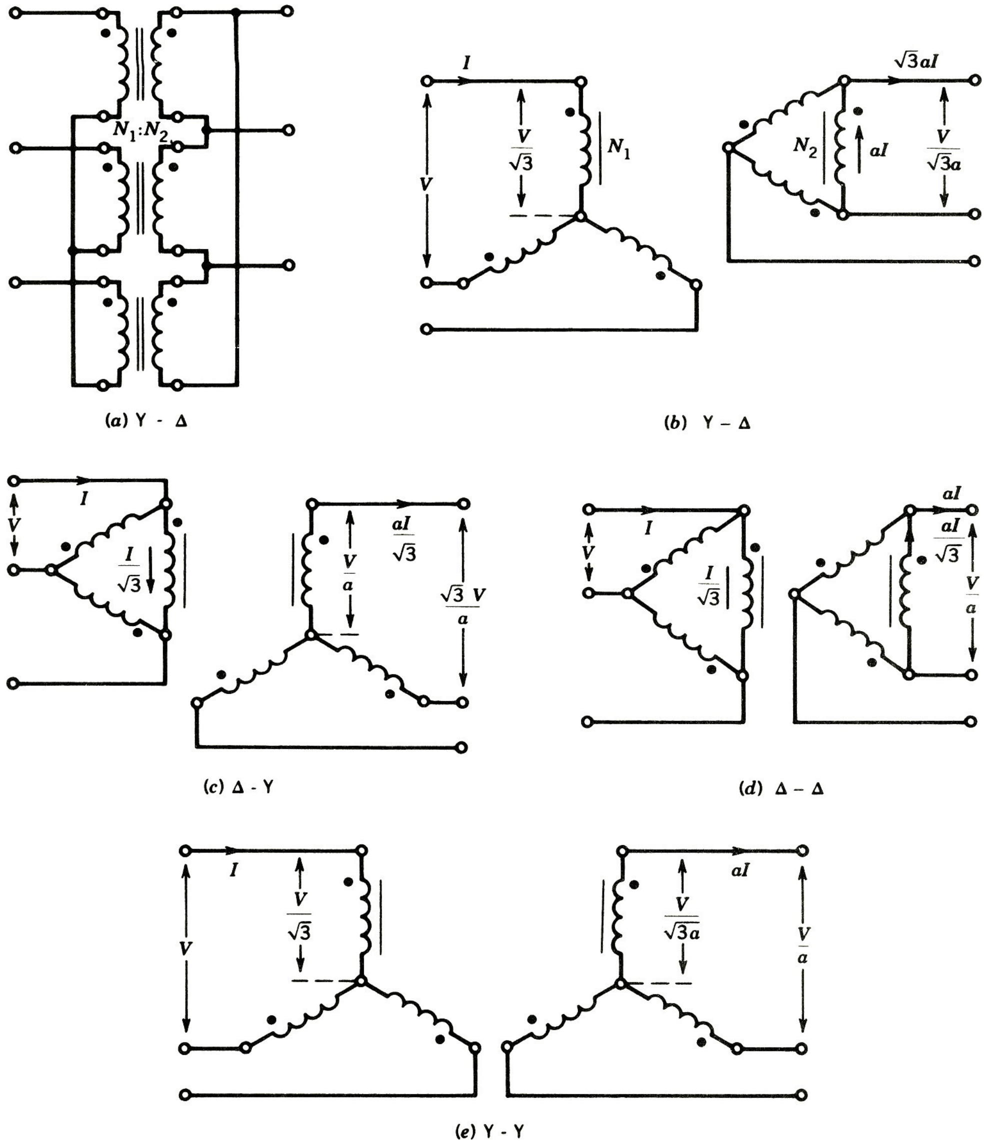


FIGURE 2.17 Three-phase transformer connections.

series. A more convenient way of showing this connection is illustrated in Fig. 2.17*b*. The primary and secondary windings shown parallel to each other belong to the same single-phase transformer. The primary and secondary voltages and currents are also shown in Fig. 2.17*b*, where V is the line-to-line voltage on the primary side and a $(= N_1/N_2)$ is the turns ratio of the single-phase transformer. Other possible connections are also shown in Figs. 2.17*c*, *d*, and *e*. It may be noted that for all possible connections, the total kVA of the three-phase transformer is shared equally by each (phase) transformer. However, the voltage and current ratings of each transformer depend on the connections used.

Y–Δ: This connection is commonly used to step down a high voltage to a lower voltage. The neutral point on the high-voltage side can be grounded, which is desirable in most cases.

Δ–Y: This connection is commonly used to step up voltage.

Δ–Δ: This connection has the advantage that one transformer can be removed for repair and the remaining two can continue to deliver three-phase power at a reduced rating of 58% of that of the original bank. This is known as the *open-delta* or *V connection*.

Y–Y: This connection is rarely used because of problems with the exciting current and induced voltages.

Phase Shift

Some of the three-phase transformer connections will result in a phase shift between the primary and secondary line-to-line voltages. Consider the phasor voltages, shown in Fig. 2.18, for the Y–Δ connections. The phasors V_{AN} and V_a are aligned, but the line voltage V_{AB} of the primary leads the line voltage V_{ab} of the secondary by 30°. It can be shown that Δ–Y connection also provides a 30° phase shift between line-to-line voltages, whereas Δ–Δ and Y–Y connections have no phase shift in their line-to-line voltages. This property of phase shift in Y–Δ or Δ–Y connections can be used advantageously in some applications.

Single-Phase Equivalent Circuit

If the three transformers are practically identical and the source and load are balanced, then the voltages and currents on both primary and secondary sides are balanced. The voltages and currents in one phase are the same as those in other phases, except that there is a phase displacement of 120°. Therefore, analysis of one phase is sufficient to determine the variables on the two sides of the transformer. A single-phase equivalent circuit can be conveniently obtained if all sources, transformer windings, and load impedances are considered to be Y-connected. The Y load can be obtained for the Δ load by the well-known Y–Δ transformation, as shown in Fig. 2.19*b*. The equivalent Y representation of the actual circuit

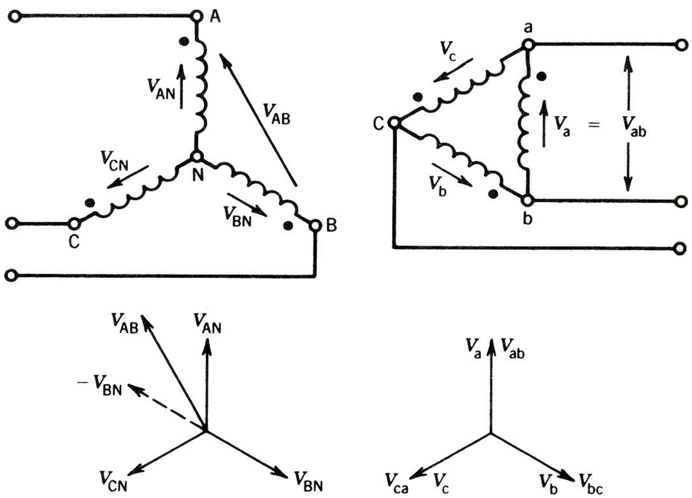


FIGURE 2.18 Phase shift in line-to-line voltages in a three-phase transformer.

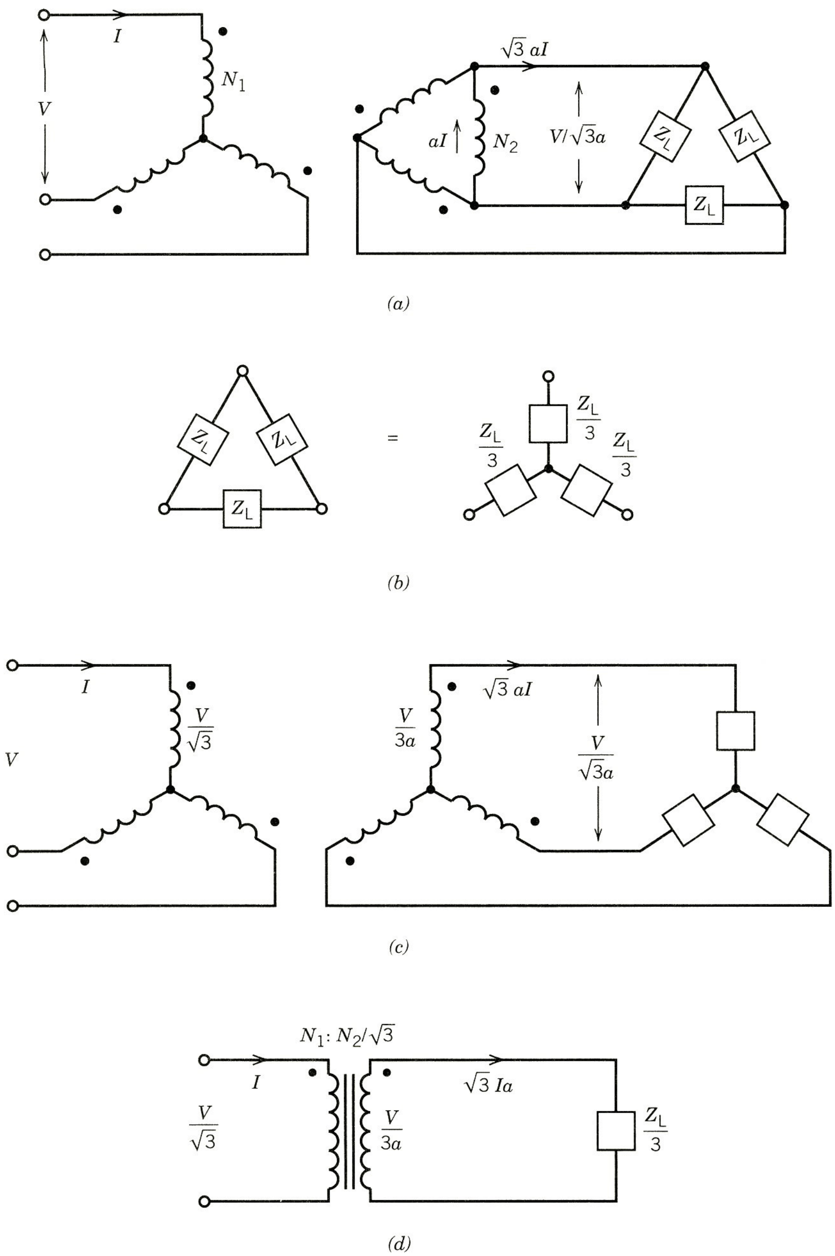


FIGURE 2.19 Three-phase transformer and equivalent circuit.

(Fig. 2.19*a*) is shown in Fig. 2.19*c*, in which the primary and secondary line currents and line-to-line voltages are identical to those of the actual circuit of Fig. 2.19*a*. The turns ratio a' of this equivalent Y–Y transformer is

$$a' = \frac{V/\sqrt{3}}{V/3a} = \sqrt{3}a \tag{2.39}$$

Also, for the actual transformer bank

$$\frac{\text{Primary line-to-line voltage}}{\text{Secondary line-to-line voltage}} = \frac{V}{V/\sqrt{3}a} = \sqrt{3}a \tag{2.40}$$

Therefore, the turns ratio for the equivalent single-phase transformer is the ratio of the line-to-line voltages on the primary and secondary sides of the actual transformer bank. The single-phase equivalent circuit is shown in Fig. 2.19*d*. This equivalent circuit will be useful if transformers are connected to load or power supply through feeders, as illustrated in Example 2.8.

EXAMPLE 2.7

Three 1ϕ, 50 kVA, 2300/230 V, 60 Hz transformers are connected to form a 3ϕ, 4000/230 V transformer bank. The equivalent impedance of each transformer referred to low voltage is $0.012 + j0.016\ \Omega$. The 3ϕ transformer supplies a 3ϕ, 120 kVA, 230 V, 0.85 PF (lag) load.

(a) Draw a schematic diagram showing the transformer connection.

(b) Determine the transformer winding currents.

(c) Determine the primary voltage (line-to-line) required.

(d) Determine the voltage regulation.

Solution

(a) The connection diagram is shown in Fig. E2.7*a*. The high-voltage windings are to be connected in wye so that the primary can be connected to the 4000 V supply. The low-voltage winding is connected in delta to form a 230 V system for the load.

(b)

$$I_s = \frac{120{,}000}{\sqrt{3} \times 230} = 301.24\ \text{A}$$

$$I_2 = \frac{301.24}{\sqrt{3}} = 173.92\ \text{A}$$

$$a = \frac{2300}{230} = 10$$

$$I_1 = \frac{173.92}{10} = 17.39\ \text{A}$$

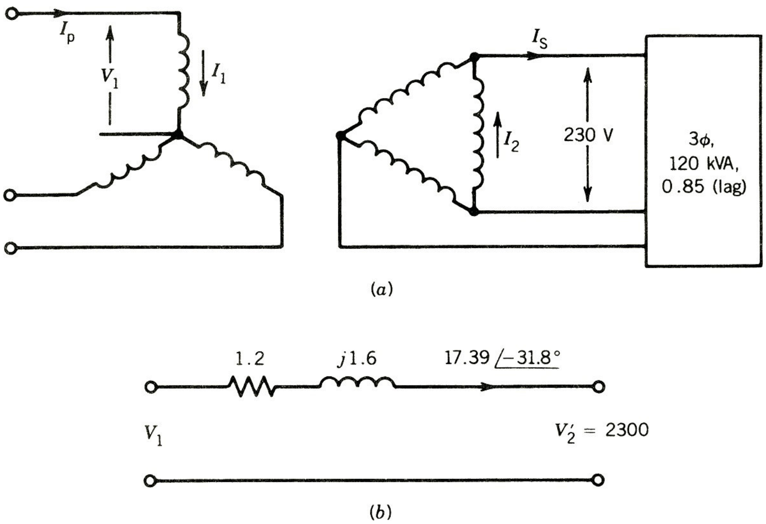


FIGURE E2.7

(c) Computation can be carried out on a per-phase basis.

$$
\begin{aligned}
Z_{eq1} &= (0.012 + j0.016)10^2 \\
&= 1.2 + j1.6\ \Omega \\
\phi &= \cos^{-1} 0.85 = 31.8^\circ
\end{aligned}
$$

The primary equivalent circuit is shown in Fig. E2.7*b*.

$$
\begin{aligned}
V_1 &= 2300\underline{/0^\circ} + 17.39\underline{/-31.8^\circ}(1.2 + j1.6) \\
|V_1| &= 2332.4\ \text{V}
\end{aligned}
$$

$$\text{Primary line-to-line voltage} = \sqrt{3}V_1 = 4039.8\ \text{V}$$

(d)

$$\text{VR} = \frac{2332.4 - 2300}{2300} \times 100\% = 1.41\% \quad \blacksquare$$

EXAMPLE 2.8

A 3ϕ, 230 V, 27 kVA, 0.9 PF (lag) load is supplied by three 10 kVA, 1330/230 V, 60 Hz transformers connected in Y–Δ by means of a common 3ϕ feeder whose impedance is $0.003 + j0.015\ \Omega$ per phase. The transformers are supplied from a 3ϕ source through a 3ϕ feeder whose impedance is $0.8 + j5.0\ \Omega$ per phase. The equivalent impedance of one transformer referred to the low-voltage side is $0.12 + j0.25\ \Omega$. Determine the required supply voltage if the load voltage is 230 V.

Solution

The circuit is shown in Fig. E2.8*a*.

The equivalent circuit of the individual transformer referred to the high-voltage side is

$$R_{\text{eqH}} + jX_{\text{eqH}} = \left(\frac{1330}{230}\right)^2 (0.12 + j0.25)$$

$$= 4.01 + j8.36$$

The turns ratio of the equivalent Y–Y bank is

$$a' = \frac{\sqrt{3} \times 1330}{230} = 10$$

The single-phase equivalent circuit of the system is shown Fig. E2.8*b*. All the impedances from the primary side can be transferred to the secondary side and combined with the feeder impedance on the secondary side.

$$R = (0.80 + 4.01)\frac{1}{10^2} + 0.003 = 0.051\ \Omega$$

$$X = (5 + 8.36)\frac{1}{10^2} + 0.015 = 0.149\ \Omega$$

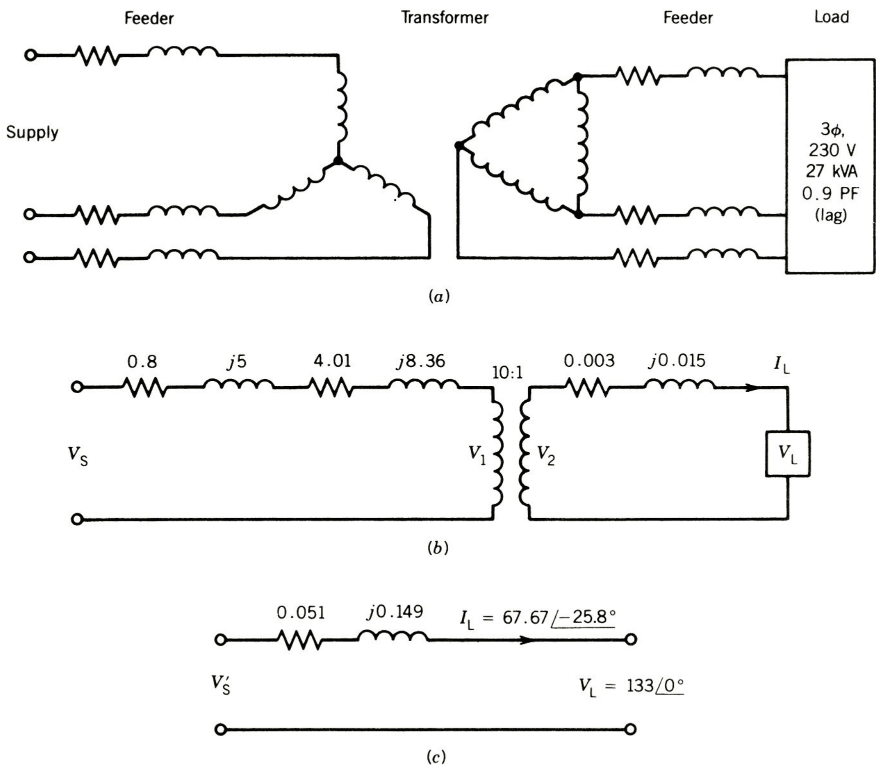


FIGURE E2.8

The circuit is shown in Fig. E2.8*c*.

$$V_{\mathrm{L}} = \frac{230}{\sqrt{3}}\underline{/0^\circ} = 133\underline{/0^\circ}\ \mathrm{V}$$

$$I_{\mathrm{L}} = \frac{27 \times 10^3}{3 \times 133} = 67.67\ \mathrm{A}$$

$$\phi_{\mathrm{L}} = -\cos^{-1}0.9 = -25.8^\circ$$

$$V'_{\mathrm{s}} = 133\underline{/0^\circ} + 67.67\underline{/-25.8^\circ}(0.051 + j0.149)$$

$$= 133\underline{/0^\circ} + 10.6571\underline{/45.3^\circ}$$

$$= 140.7\underline{/3.1^\circ}\ \mathrm{V}$$

$$V_{\mathrm{s}} = 140.7 \times 10 = 1407\ \mathrm{V}$$

The line-to-line supply voltage is

$$1407\sqrt{3} = 2437\ \mathrm{V} \quad \blacksquare$$

V Connection

It was stated earlier that in the Δ–Δ connection of three single-phase transformers, one transformer can be removed and the system can still deliver three-phase power to a three-phase load. This configuration is known as an open-delta or V connection. It may be employed in an emergency situation when one transformer must be removed for repair and continuity of service is required.

Consider Fig. 2.20*a*, in which one transformer, shown dotted, is removed. For simplicity the load is considered to be Y-connected. Figure 2.20*b* shows the phasor diagram for voltages and currents. Here V_{AB}, V_{BC}, and V_{CA} represent the line-to-line voltages of the primary; V_{ab}, V_{bc}, and V_{ca} represent the line-to-line voltages of the secondary; and V_{an}, V_{bn}, and V_{cn} represent the phase voltages of the load. For an inductive load, the load currents I_a, I_b, and I_c will lag the corresponding voltages V_{an}, V_{bn}, and V_{cn} by the load phase angle ϕ.

Transformer windings ab and bc deliver power

$$P_{ab} = V_{ab}I_a \cos(30 + \phi) \tag{2.41}$$

$$P_{bc} = V_{cb}I_c \cos(30 - \phi) \tag{2.42}$$

Let

$|V_{ab}| = |V_{cb}| = V$, voltage rating of the transformer secondary winding
$|I_a| = |I_c| = I$, current rating of the transformer secondary winding

and $\phi = 0$ for a resistive load. Power delivered to the load by the V connection is

$$P_v = P_{ab} + P_{bc} = 2VI\cos 30^\circ \tag{2.43}$$

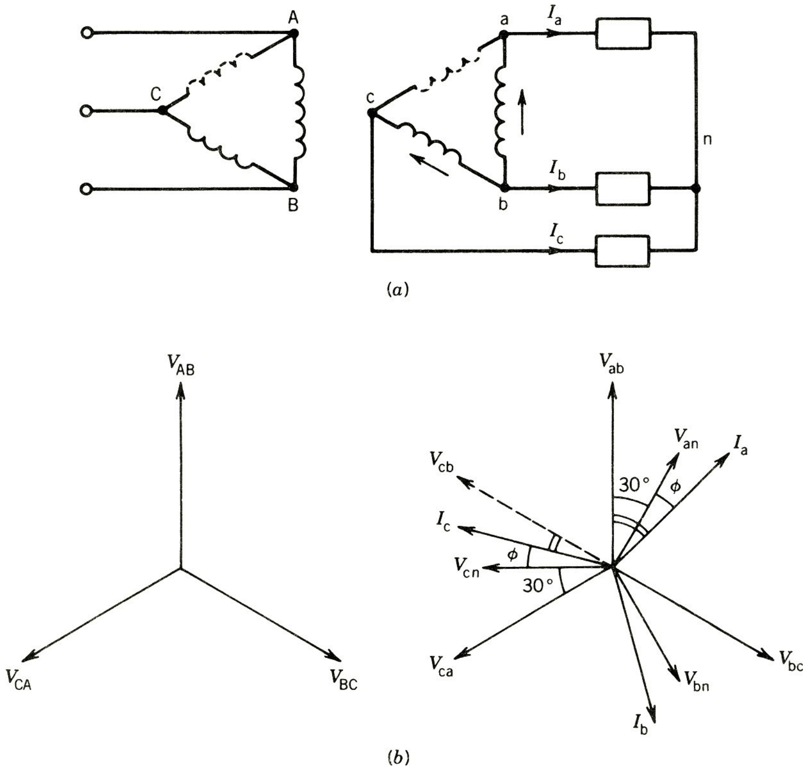


FIGURE 2.20 V connection.

With all three transformers connected in delta, the power delivered is

$$P_\Delta = 3VI \tag{2.44}$$

From Eqs. 2.43 and 2.44,

$$\frac{P_V}{P_\Delta} = \frac{2\cos 30^\circ}{3} = 0.58 \tag{2.45}$$

The V connection is capable of delivering 58% power without overloading the transformer (i.e., not exceeding the current rating of the transformer windings).

2.6.2 THREE-PHASE TRANSFORMER ON A COMMON MAGNETIC CORE (THREE-PHASE UNIT TRANSFORMER)

A three-phase transformer can be constructed by having three primary and three secondary windings on a common magnetic core. Consider three single-phase core-type units as shown in Fig. 2.21*a*. For simplicity, only the primary windings have been shown. If balanced three-phase sinusoidal voltages are applied to the windings, the fluxes Φ_a, Φ_b, and Φ_c will also be sinusoidal

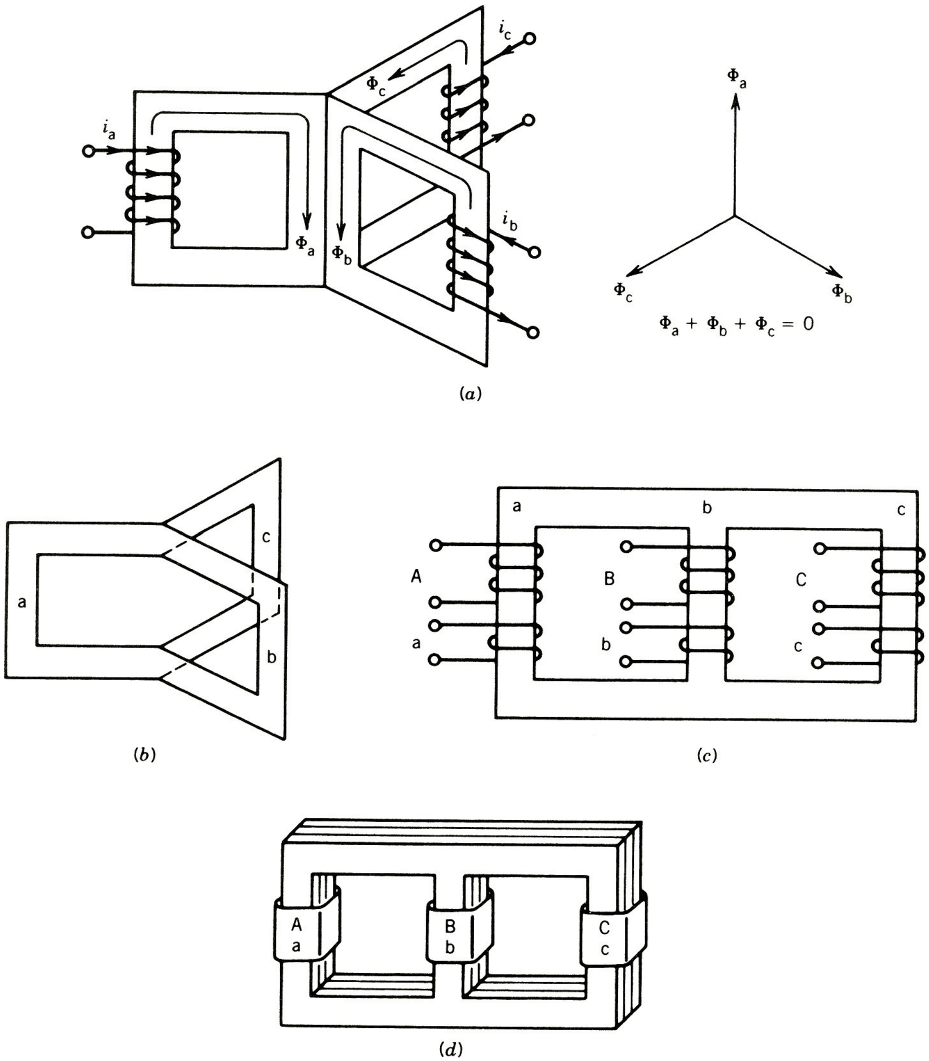


FIGURE 2.21 Development of a three-phase core-type transformer.

and balanced. If the three legs carrying these fluxes are merged, the net flux in the merged leg is zero. This leg can therefore be removed as shown in Fig. 2.21*b*. This structure is not convenient to build. However, if section b is pushed in between sections a and c by removing its yokes, a common magnetic structure, shown in Fig. 2.21*c*, is obtained. This core structure can be built using stacked laminations as shown in Fig. 2.21*d*. Both primary and secondary windings of a phase are placed on the same leg. Note that the magnetic paths of legs a and c are somewhat longer than that of leg b (Fig. 2.21*c*). This will result in some imbalance in the magnetizing currents. However, this imbalance is not significant.

Figure 2.22 shows a picture of a three-phase transformer of this type. Such a transformer weighs less, costs less, and requires less space than a three-phase transformer bank of the same

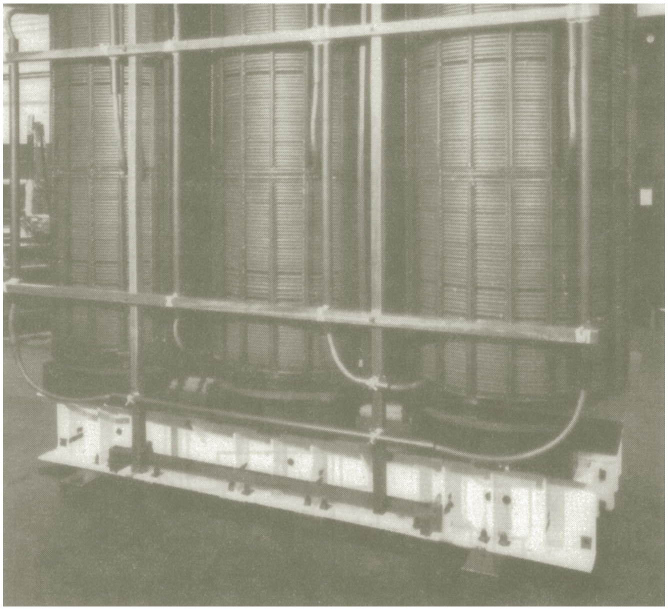


FIGURE 2.22 Photograph of a 3ϕ unit transformer.

rating. The disadvantage is that if one phase breaks down, the whole transformer must be removed for repair.

2.7 HARMONICS IN THREE-PHASE TRANSFORMER BANKS

If a transformer is operated at a higher flux density, it will require less magnetic material. Therefore, from an economic point of view, a transformer is designed to operate in the saturating region of the magnetic core. This makes the exciting current nonsinusoidal, as discussed in Chapter 1. The exciting current will contain the fundamental and all odd harmonics. However, the third harmonic is the predominant one, and for all practical purposes harmonics higher than third (fifth, seventh, ninth, etc.) can be neglected. At rated voltage the third harmonic in the exciting current can be 5 to 10% of the fundamental. At 150% rated voltage, the third harmonic current can be as high as 30 to 40% of the fundamental.

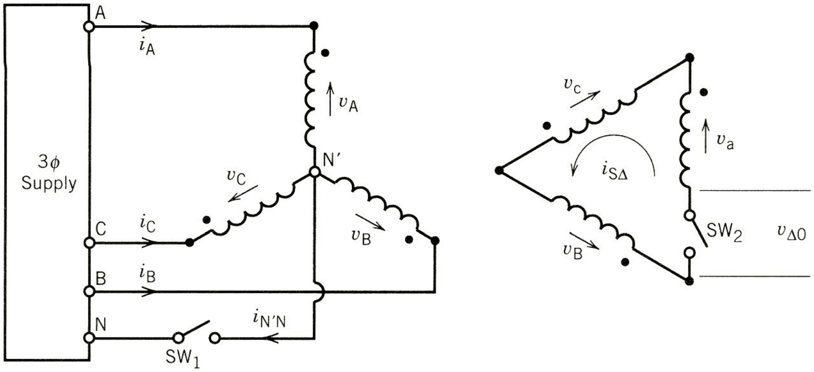
In this section we will study how these harmonics are generated in various connections of the three-phase transformers and ways to limit their effects.

Consider the system shown in Fig. 2.23*a*. The primary windings are connected in Y and the neutral point N of the supply is available. The secondary windings can be connected in Δ.

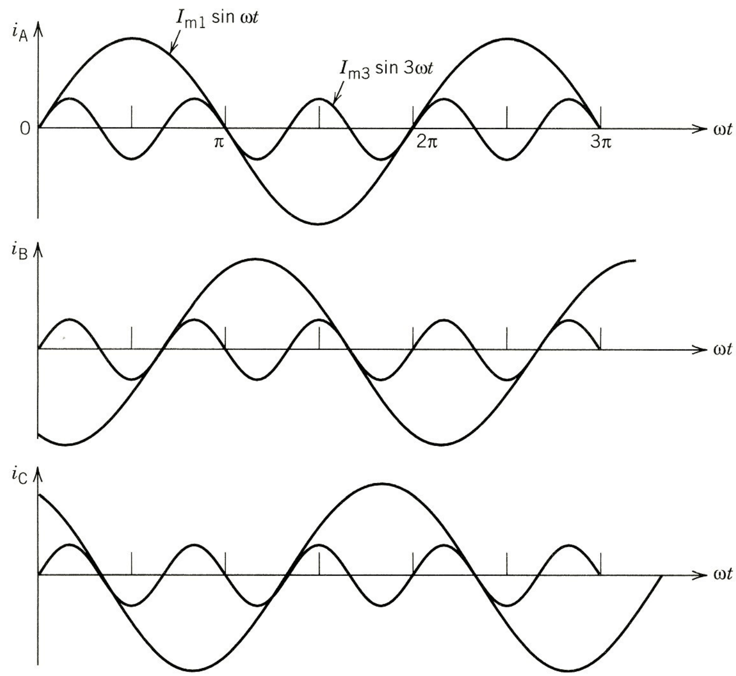
Switch SW_1 Closed and Switch SW_2 Open

Because SW_2 is open, no current flows in the secondary windings. The currents flowing in the primary are the exciting currents. We assume that the exciting currents contain only fundamental and third-harmonic currents as shown in Fig. 2.23*b*. Mathematically,

$$i_A = I_{m1} \sin \omega t + I_{m3} \sin 3\,\omega t \tag{2.46}$$



(a)



(b)

FIGURE 2.23 Harmonic current in three-phase transformer connections. (*a*) Y–Δ connection, (*b*) Waveforms of exciting currents.

$$i_B = I_{m1} \sin(\omega t - 120^\circ) + I_{m3} \sin 3(\omega t - 120^\circ) \tag{2.47}$$

$$i_C = I_{m1} \sin(\omega t - 240^\circ) + I_{m3} \sin 3(\omega t - 240^\circ) \tag{2.48}$$

The current in the neutral line is

$$i_{N'N} = i_A + i_B + i_C = 3I_{m3} \sin 3\omega t \tag{2.49}$$

Note that fundamental currents in the windings are phase-shifted by 120° from each other, whereas third-harmonic currents are all in phase. The neutral line carries only the third-harmonic current, as can be seen in the oscillogram of Fig. 2.24*a*.

Because the exciting current is nonsinusoidal (Fig. 2.24*b*), the flux in the core and hence the induced voltages in the windings will be sinusoidal. The secondary windings are open, and therefore the voltage across a secondary winding will represent the induced voltage.

$$v_{\Delta 0} = v_a + v_b + v_c = 0 \tag{2.50}$$

Both SW_1 and SW_2 Open

In this case the third-harmonic currents cannot flow in the primary windings. Therefore, the primary currents are essentially sinusoidal. If the exciting current is sinusoidal, the flux is nonsinusoidal because of nonlinear $B-H$ characteristics of the magnetic core, and it contains third-harmonic components. This will induce third-harmonic voltage in the windings. The phase voltages are therefore nonsinusoidal, containing fundamental and third-harmonic voltages.

$$v_A = v_{A1} + v_{A3} \tag{2.51}$$

$$v_B = v_{B1} + v_{B3} \tag{2.52}$$

$$v_C = \underbrace{v_{C1}}_{\text{fundamental voltages}} + \underbrace{v_{C3}}_{\text{third-harmonic voltages}} \tag{2.53}$$

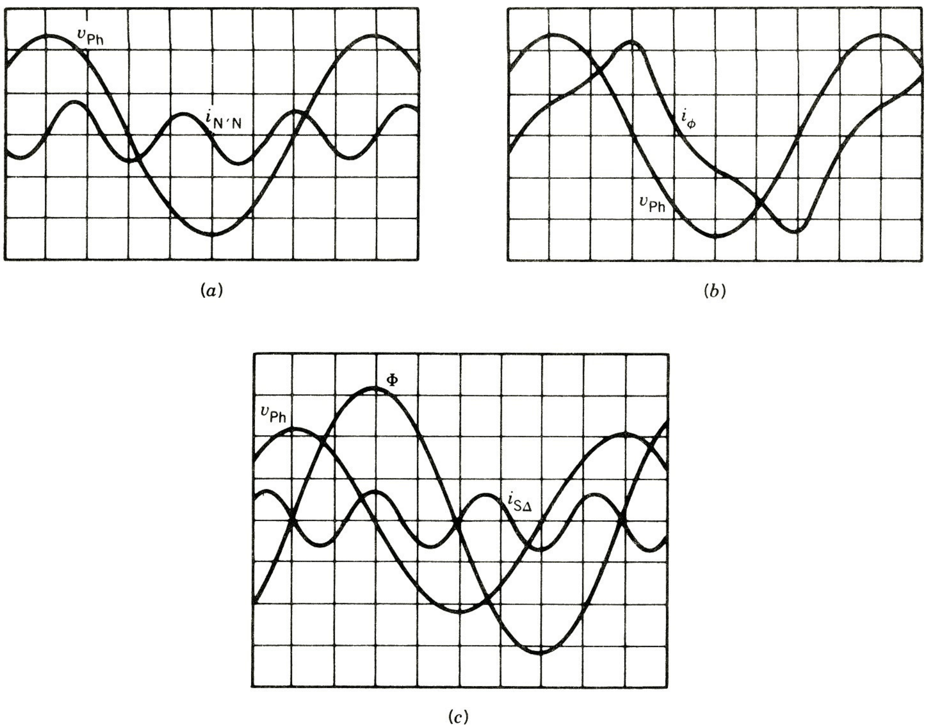


FIGURE 2.24 Oscillograms of currents and voltages in a Y–Δ-connected transformer.

The line-to-line voltage is

$$v_{AB} = v_A - v_B \tag{2.54}$$

$$= v_{A1} - v_{B1} + v_{A3} - v_{B3} \tag{2.55}$$

Because v_{A3} and v_{B3} are in phase and have the same magnitude,

$$v_{A3} - v_{B3} = 0 \tag{2.56}$$

Therefore,

$$v_{AB} = v_{A1} - v_{B1} \tag{2.57}$$

Note that although phase voltages have third-harmonic components, the line-to-line voltages do not.

The open-delta voltage (Fig. 2.23*a*) of the secondary is

$$v_{\Delta 0} = v_a + v_b + v_c \tag{2.58}$$

$$= (v_{a1} + v_{b1} + v_{c1}) + (v_{a3} + v_{b3} + v_{c3}) \tag{2.58a}$$

$$= v_{a3} + v_{b3} + v_{c3} \tag{2.58b}$$

$$= 3v_{a3} \tag{2.58c}$$

The voltage across the open delta is the sum of the three third-harmonic voltages induced in the secondary windings.

Switch SW_1 Open and Switch SW_2 Closed

If switch SW_2 is closed, the voltage $v_{\Delta 0}$ will drive a third-harmonic current around the secondary delta. This will provide the missing third-harmonic component of the primary exciting current and consequently the flux and induced voltage will be essentially sinusoidal, as shown in Fig. 2.24*c*.

Y–Y System with Tertiary (Δ) Winding

For high voltages on both sides, it may be desirable to connect both primary and secondary windings in Y, as shown in Fig. 2.25. In this case third-harmonic currents cannot flow either in primary or in secondary. A third set of windings, called a *tertiary winding*, connected in Δ is normally fitted on the core so that the required third-harmonic component of the exciting current can be supplied. This tertiary winding can also supply an auxiliary load if necessary.

2.8 PER-UNIT (PU) SYSTEM

Computations using the actual values of parameters and variables may be lengthy and time-consuming. However, if the quantities are expressed in a per-unit (pu) system, computations

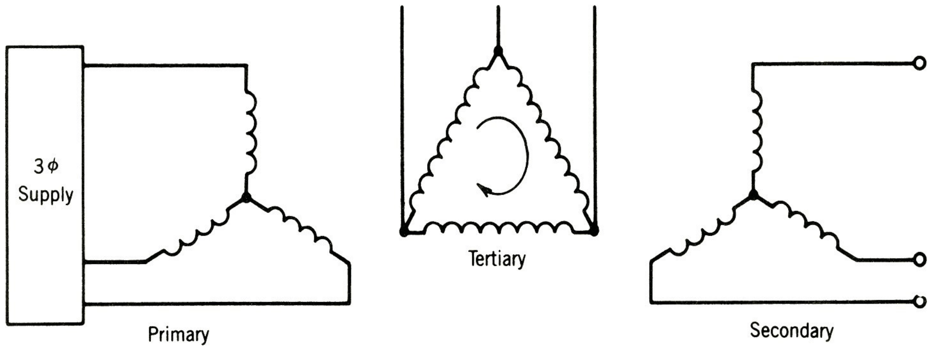


FIGURE 2.25 Y–Y system with a tertiary (Δ) transformer.

are much simplified. The pu quantity is defined as follows:

$$\text{Quantity in pu} = \frac{\text{actual quantity}}{\text{base (or reference) value of the quantity}} \tag{2.59}$$

There are two major advantages in using a per-unit system: (1) The parameters and variables fall in a narrow numerical range when expressed in a per-unit system; this simplifies computations and makes it possible to quickly check the correctness of the computed values. (2) One need not refer circuit quantities from one side to another; therefore, a common source of mistakes is removed.

To establish a per-unit system, it is necessary to select base (or reference) values for any two of power, voltage, current, and impedance. Once base values for any two of the four quantities have been selected, the base values for the other two can be determined from the relationship among these four quantities. Usually base values of power and voltage are selected first and base values of current and impedance are obtained as follows:

$$P_{\text{base}}, V_{\text{base}} \text{ selected}$$

$$I_{\text{base}} = \frac{P_{\text{base}}}{V_{\text{base}}} \tag{2.60}$$

$$Z_{\text{base}} = \frac{V_{\text{base}}}{I_{\text{base}}} \tag{2.61}$$

$$= \frac{V_{\text{base}}^2}{P_{\text{base}}} \tag{2.62}$$

Although base values can be chosen arbitrarily, normally the rated volt-amperes and rated voltage are taken as the base values for power and voltage, respectively.

$$S_{\text{base}} = P_{\text{base}} = \text{rated volt-amperes (VA)}$$

$$V_{\text{base}} = \text{rated voltage } (V)$$

In the case of a transformer, the power base is the same for both primary and secondary. However, the values of V_{base} are different on each side, because rated voltages are different for the two sides.

Primary side:

$$V_{\text{base}}, V_{\text{B1}} = V_{\text{R1}} = \text{rated voltage of primary}$$
$$I_{\text{base}}, I_{\text{B1}} = I_{\text{R1}} = \text{rated current of primary}$$
$$Z_{\text{base}}, Z_{\text{B1}} = \frac{V_{\text{R1}}}{I_{\text{R1}}}$$

Let

$$Z_{\text{eq1}} = \text{equivalent impedance of the transformer referred to the primary side}$$
$$Z_{\text{eq1,pu}} = \text{per-unit value of } Z_{\text{eq1}} = Z_{\text{eq1}}/Z_{\text{B1}}$$
$$= \frac{Z_{\text{eq1}}}{V_{\text{R1}}/I_{\text{R1}}} \tag{2.63}$$
$$= Z_{\text{eq1}} \frac{I_{\text{R1}}}{V_{\text{R1}}}$$

Secondary side:

$$V_{\text{base}}, V_{\text{B2}} = V_{\text{R2}} = \text{rated voltage of secondary}$$
$$I_{\text{base}}, I_{\text{B2}} = I_{\text{R2}} = \text{rated current of secondary}$$
$$Z_{\text{base}}, Z_{\text{B2}} = \frac{V_{\text{R2}}}{I_{\text{R2}}}$$

Let

$$Z_{\text{eq2}} = \text{equivalent impedance referred to the secondary side}$$
$$Z_{\text{eq2,pu}} = \text{per-unit value of } Z_{\text{eq2}}$$
$$= \frac{Z_{\text{eq2}}}{Z_{\text{B2}}} \tag{2.64}$$
$$= \frac{Z_{\text{eq1}}/a^2}{Z_{\text{B1}}/a^2}$$
$$= \frac{Z_{\text{eq1}}}{Z_{\text{B1}}} \tag{2.65}$$
$$Z_{\text{eq2,pu}} = Z_{\text{eq1,pu}} \tag{2.66}$$

Therefore, the per-unit transformer impedance is the same referred to either side of the transformer. This is another advantage of expressing quantities in a per-unit system.

In a transformer, when voltages or currents of either side are expressed in a per-unit system, they have the same per-unit values.

$$I_{1,\mathrm{pu}} = \frac{I_1}{I_{\mathrm{B1}}} = \frac{I_1}{I_{\mathrm{R1}}} = \frac{I_2/a}{I_{\mathrm{R2}}/a} = \frac{I_2}{I_{\mathrm{R2}}} = \frac{I_2}{I_{\mathrm{B2}}} = I_{2,\mathrm{pu}} \tag{2.67}$$

$$\begin{aligned} V_{1,\mathrm{pu}} &= \frac{V_1}{V_{\mathrm{B1}}} = \frac{V_1}{V_{\mathrm{R1}}} = \frac{aV_2}{aV_{\mathrm{R2}}} \\ &= \frac{V_2}{V_{\mathrm{R2}}} = \frac{V_2}{V_{\mathrm{B2}}} = V_{2,\mathrm{pu}} \end{aligned} \tag{2.68}$$

2.8.1 TRANSFORMER EQUIVALENT CIRCUIT IN PER-UNIT FORM

The equivalent circuit of a transformer referred to the primary side is shown in Fig. 2.26*a*. The equation in terms of actual values is

$$V_1 = I_1 Z_{\mathrm{eq1}} + V_2' \tag{2.69}$$

The equation in per-unit form can be obtained by dividing Eq. 2.69 throughout by the base value of the primary voltage.

$$\begin{aligned} \frac{V_1}{V_{\mathrm{R1}}} &= \frac{I_1 Z_{\mathrm{eq1}}}{V_{\mathrm{R1}}} + \frac{V_2'}{V_{\mathrm{R1}}} \\ &= \frac{I_1 Z_{\mathrm{eq1}}}{I_{\mathrm{R1}} Z_{\mathrm{B1}}} + \frac{\mathrm{a}V_2}{\mathrm{a}V_{\mathrm{R2}}} \end{aligned}$$

$$V_{1,\mathrm{pu}} = I_{1,\mathrm{pu}} Z_{\mathrm{eq1,pu}} + V_{2,\mathrm{pu}} \tag{2.70}$$

Based on Eq. 2.70, the equivalent circuit in per-unit form is shown in Fig. 2.26*b*. It has been shown that the voltages, currents, and impedances in per-unit representation have the same

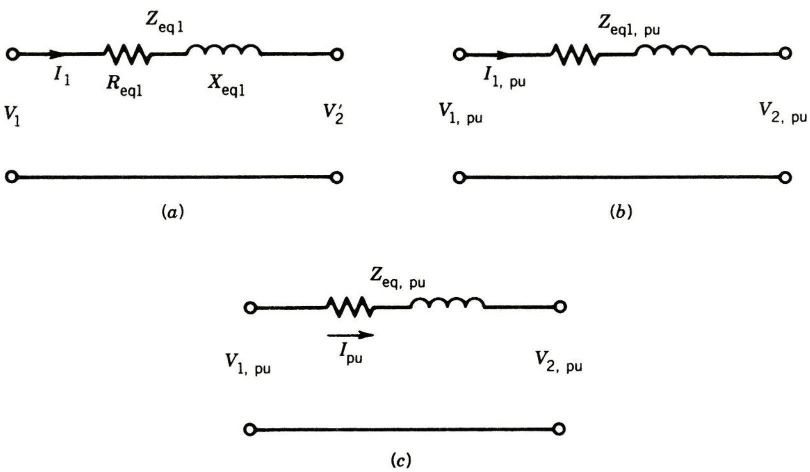


FIGURE 2.26 Transformer equivalent circuit in per-unit form.

values whether they are referred to primary or secondary. Hence, the transformer equivalent circuit in per-unit form for either side is the one shown in Fig. 2.26*c*. Note that the values of $V_{1,\text{pu}}$ and $V_{2,\text{pu}}$ are generally close to 1 pu, and this makes the analysis somewhat easier.

2.8.2 FULL-LOAD COPPER LOSS

Let

$$\begin{aligned} P_{\text{cu,FL}} &= \text{full-load copper loss} \\ &= I_{\text{R1}}^2 R_{\text{eq1}} \end{aligned} \tag{2.71}$$

The full-load copper loss in per-unit form based on the volt-ampere rating of the transformer is

$$\begin{aligned} P_{\text{cu,FL}}\big|_{\text{pu}} &= \frac{I_{\text{R1}}^2 R_{\text{eq1}}}{P_{\text{base}}} \\ &= \frac{I_{\text{R1}}^2 R_{\text{eq1}}}{V_{\text{R1}} I_{\text{R1}}} \\ &= \frac{R_{\text{eq1}}}{V_{\text{R1}}/I_{\text{R1}}} \\ &= \frac{R_{\text{eq1}}}{Z_{\text{B1}}} \\ &= R_{\text{eq1,pu}} \end{aligned} \tag{2.72}$$

Hence, the transformer resistance expressed in per-unit form also represents the full-load copper loss in per-unit form. The per-unit value of the resistance is therefore more useful than its ohmic value in determining the performance of a transformer.

EXAMPLE 2.9

The exciting current of a 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer is 0.25 A when measured on the high-voltage side. Its equivalent impedance is $10.4 + j31.3\ \Omega$ when referred to the high-voltage side. Taking the transformer rating as base,

(a) Determine the base values of voltages, currents, and impedances for both high-voltage and low-voltage sides.

(b) Express the exciting current in per-unit form for both high-voltage and low-voltage sides.

(c) Obtain the equivalent circuit in per-unit form.

(d) Find the full-load copper loss in per-unit form.

(e) Determine the per-unit voltage regulation (using the per-unit equivalent circuit from part c) when the transformer delivers 75% full load at 0.6 lagging power factor.

Solution

(a) $P_{\text{base}} = 10{,}000\ \text{VA}, a = 10.$

Using the subscripts H and L to indicate high-voltage and low-voltage sides, the base values of voltages, currents, and impedances are

$$V_{\text{base,H}} = 2200\ \text{V} = 1\ \text{pu}$$

$$V_{\text{base,L}} = 220\ \text{V} = 1\ \text{pu}$$

$$I_{\text{base,H}} = \frac{10{,}000}{2200} = 4.55\ \text{A} = 1\ \text{pu}$$

$$I_{\text{base,L}} = \frac{10{,}000}{220} = 45.5\ \text{A} = 1\ \text{pu}$$

$$Z_{\text{base,H}} = \frac{2200}{4.55} = 483.52\ \Omega = 1\ \text{pu}$$

$$Z_{\text{base,L}} = \frac{220}{45.5} = 4.835\ \Omega = 1\ \text{pu}$$

(b)

$$I_{\phi\text{H}}\big|_{\text{pu}} = \frac{0.25}{4.55} = 0.055\ \text{pu}$$

The exciting current referred to the low-voltage side is $0.25 \times 10 = 2.5$ A. Its per-unit value is

$$I_{\phi\text{L}}\big|_{\text{pu}} = \frac{2.5}{45.5} = 0.055\ \text{pu}$$

Note that although the actual values of the exciting current are different for the two sides, the per-unit values are the same. For this transformer, this means that the exciting current is 5.5% of the rated current of the side in which it is measured.

(c)

$$Z_{\text{eq,H}}\big|_{\text{pu}} = \frac{10.4 + j31.3}{483.52} = 0.0215 + j0.0647\ \text{pu}$$

The equivalent impedance referred to the low-voltage side is

$$Z_{\text{eq,L}} = (10.4 + j31.3)\frac{1}{100}$$

$$= 0.104 + j0\ .313\ \Omega$$

Its per-unit value is

$$Z_{\text{eq,1}}\big|_{\text{pu}} = \frac{0.104 + j0.313}{4.835} = 0.0215 + j0.0647\ \text{pu}$$

The per-unit values of the equivalent impedances referred to the high-and low-voltage sides are the same. The per-unit equivalent circuit is shown in Fig. E2.9.

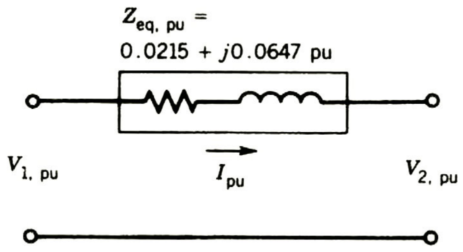


FIGURE E2.9

(d)

$$P_{cu,FL} = 4.55^2 \times 10.4 \text{ W}$$
$$= 215 \text{ W}$$
$$P_{cu,FL}\big|_{pu} = \frac{215}{10{,}000} = 0.0215 \text{ pu}$$

Note that this is same as the per-unit value of the equivalent resistance.

(e) From Fig. E2.9

$$I = 0.75\underline{/-53.13^\circ} \text{ pu}$$
$$V_2 = 1\underline{/0^\circ} \text{ pu}$$
$$Z_{eq,pu} = 0.0215 + j0.0647 \text{ pu}$$
$$V_1 = 1\underline{/0^\circ} + 0.75\underline{/-53.13^\circ}(0.0215 + j0.0647)$$
$$= 1.0486\underline{/9^\circ} \text{ pu}$$
$$\text{Voltage regulation} = \frac{1.0486 - 1.0}{1.0} = 0.0486 \text{ pu}$$
$$= 4.86\% \qquad \text{(see Example 2.3)}$$

Note that the computation in the per-unit system involves smaller numerical values than the computation using actual values (see Example 2.3). Also, the value of V_1 in pu form promptly gives a perception of voltage regulation. ■

PROBLEMS

2.1 A 1 φ, 4600/460 V, 60 Hz, transformer is connected to a 1φ, 60 Hz, 4600 V power supply. The maximum flux density in the core is 0.85 T. If the induced voltage per turn in 10 V, determine

(a) The primary turns (N_1) and the secondary turns (N_2).

(b) The cross-sectional area (A_c) of the core.

2.2 The flux in the core of a 1 φ transformer varies with time as shown in Fig. P2.2. The primary coil has 400 turns and the secondary coil has 100 turns. Sketch the waveform of the induced voltage e_1 in the primary winding.

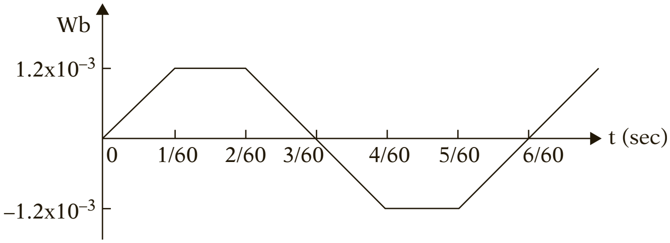


FIGURE P2.2

2.3 A single-phase transformer has 800 turns on the primary winding and 400 turns on the secondary winding. The cross-sectional core area is 50 cm^2. If the primary winding is connected to a 1 φ, 1000 V, 60 Hz supply, calculate

(a) The maximum value of the flux density in the core, B_{max}.

(b) The induced voltage in the secondary winding.

2.4 A single-phase transformer has 500 turns in the primary winding. When it is connected to a 1 φ, 120 V, 60 Hz power supply, the no-load current is 1.6 A and the no-load power is 80 W. Neglect the winding resistance and leakage reactance of the winding. Calculate

(a) The core loss current, I_{c}.

(b) The magnetizing current, I_{m}.

(c) The peak value of the core flux, Φ_{max}.

(d) The magnetizing impedance Z_{m}, magnetizing reactance X_{m}, core loss resistance R_{c}.

2.5 A coil with 100 turns is connected to a 120 V, 60 Hz power supply. If the magnetizing current is 5 A, calculate the following:

(a) The peak value of the flux, Φ_{max}.

(b) The peak value of the mmf, MMF_{peak}.

(c) The reactance of the coil, X_{m}.

(d) The inductance of the coil, L_{m}.

2.6 A 1 φ, 5 kVA, 240/120 V, 60 Hz transformer has a core loss of 100 W at rated voltage and copper loss of 120 W at rated current. Calculate the efficiency for the following load condition:

(a) It is delivering 5 kVA at rated voltage and a power factor of 0.8.

(b) It is delivering 2 kVA at rated voltage and a power factor of 0.8.

2.7 A 1 φ, 1200 kVA, 240/120 V, 60 Hz transformer has a no load loss of 3.2 kW at rated voltage and a copper loss of 9.5 kW at rated current. Determine the efficiency for the following load conditions:

(a) 1200 kVA at unity power factor.

(b) 1200 kVA at 0.9 power factor.

(c) 1200 kVA at 0.0 power factor (i.e., pure L or C load).

2.8 A resistive load varies from 1 to 0.5 Ω. The load is supplied by an ac generator through an ideal transformer whose turns ratio can be changed by using different taps as shown in Fig. P2.8. The generator can be modeled as a constant voltage of 100 V (rms) in series with an inductive reactance of $j1\ \Omega$. For maximum power transfer to the load, the effective load resistance seen at the transformer primary (generator side) must equal the series impedance of the generator—that is, the referred value of R to the primary side is always 1 Ω.

(a) Determine the range of turns ratio for maximum power transfer to the load.

(b) Determine the range of load voltages for maximum power transfer.

(c) Determine the power transferred.

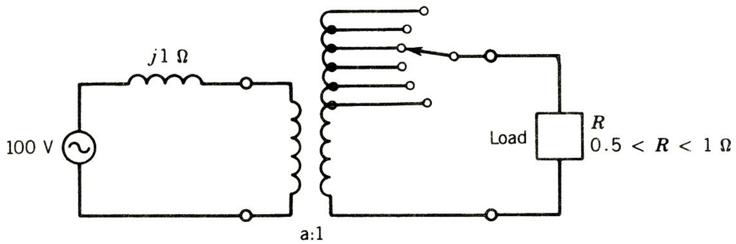


FIGURE P2.8

2.9 A 1ϕ, two-winding transformer has 1000 turns on the primary and 500 turns on the secondary. The primary winding is connected to a 220 V supply and the secondary winding is connected to a 5 kVA load. The transformer can be considered ideal.

(a) Determine the load voltage.

(b) Determine the load impedance.

(c) Determine the load impedance referred to the primary.

2.10 A 1ϕ, 10 kVA, 220/110 V, 60 Hz transformer is connected to a 220 V supply. It draws rated current at 0.8 power factor leading. The transformer may be considered ideal.

(a) Determine the kVA rating of the load.

(b) Determine the impedance of the load.

2.11 For the circuit shown in Fig. P2.11, consider the transformer to be ideal with the turns ratio 1:100. Calculate the actual load voltage V_2 and the supply current I_1.

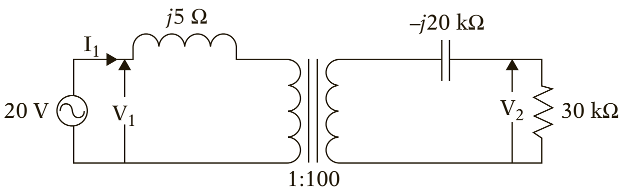


FIGURE P2.11

2.12 A 1 φ, 2400/240 V transformer has $R_1 = 0.75\,\Omega$ and $X_1 = 1.5\,\Omega$. It is drawing 100 A at a lagging power factor of 0.8. Determine the induced voltage E_1 in the primary winding.

2.13 A 1ϕ, 100 kVA, 1000/100 V transformer gave the following test results:

open-circuit test (HV side open)

100 V, 6.0 A, 400 W

short-circuit test

50 V, 100 A, 1800 W

(a) Determine the rated voltage and rated current for the high-voltage and low-voltage sides.

(b) Derive an approximate equivalent circuit referred to the HV side.

(c) Determine the voltage regulation at full load, 0.6 PF leading.

(d) Draw the phasor diagram for condition (c).

2.14 A 1ϕ, 25 kVA, 220/440 V, 60 Hz transformer gave the following test results.

Open circuit test (440 V side open): 220 V, 9.5 A, 650 W

Short-circuit test (220 V side shorted): 37.5 V, 55 A, 950 W

(a) Derive the approximate equivalent circuit in per-unit values.

(b) Determine the voltage regulation at full load, 0.8 PF lagging.

(c) Draw the phasor diagram for condition (b).

2.15 A 1ϕ, 10 kVA, 2400/120 V, 60 Hz transformer has the following equivalent circuit parameters:

$$Z_{\text{eq,H}} = 5 + j25\,\Omega$$
$$R_{\text{c(HV)}} = 64\,\text{k}\,\Omega$$
$$X_{\text{M(HV)}} = 9.6\,\text{k}\,\Omega$$

Standard no-load and short-circuit tests are performed on this transformer.
Determine the following:

No-load test results :	V_{oc}, I_{oc}, and P_{oc}
Short-circuit test results :	V_{sc}, I_{sc}, and P_{sc}

2.16 A 1ϕ, 100 kVA, 11,000/2200 V, 60 Hz transformer has the following parameters.

$$R_{\text{HV}} = 6.0\,\Omega$$
$$L_{\text{HV}} = 0.08\,\text{H}$$
$$L_{m(\text{HV})} = 160\,\text{H}$$
$$R_{c(\text{HV})} = 125\,\text{k}\Omega$$
$$R_{\text{LV}} = 0.28\,\Omega$$
$$L_{\text{LV}} = 0.0032\,\text{H}$$

Obtain an equivalent circuit of the transformer:

(a) Referred to the high-voltage side.

(b) Referred to the low-voltage side.

2.17 A 1ϕ, 440 V, 80 kW load, having a lagging power factor of 0.8, is supplied through a feeder of impedance $0.6 + j1.6\,\Omega$ and a 1ϕ, 100 kVA, 220/440 V, 60 Hz transformer. The equivalent impedance of the transformer referred to the high-voltage side is $1.15 + j4.5\,\Omega$.

(a) Draw the schematic diagram showing the transformer connection.

(b) Determine the voltage at the high-voltage terminal of the transformer.

(c) Determine the voltage at the sending end of the feeder.

2.18 A 1ϕ, 3 kVA, 240/120 V, 60 Hz transformer has the following parameters:

$$R_{\text{HV}} = 0.25\,\Omega, \qquad R_{\text{LV}} = 0.05\,\Omega$$
$$X_{\text{HV}} = 0.75\,\Omega, \qquad X_{\text{LV}} = 0.18\,\Omega$$

(a) Determine the voltage regulation when the transformer is supplying full load at 110 V and 0.9 leading power factor.

(b) If the load terminals are accidentally short-circuited, determine the currents in the high-voltage and low-voltage windings.

2.19 A single-phase, 300 kVA, 11 kV/2.2 kV, 60 Hz transformer has the following equivalent circuit parameters referred to the high-voltage side:

$$R_{c(\text{HV})} = 57.6\,\text{k}\Omega, \qquad X_{m(\text{HV})} = 16.34\,\text{k}\Omega$$
$$R_{\text{eq(HV)}} = 2.784\,\Omega \qquad X_{\text{eq(HV)}} = 8.45\,\Omega$$

(a) Determine

(i) No-load current as a percentage of full-1ϕ load current.

(ii) No-load power loss (i.e., core loss).

(iii) No-load power factor.

(iv) Full-load copper loss.

(b) If the load impedance on the low-voltage side is $Z_{\text{load}} = 16\underline{/60^\circ}\,\Omega$ determine the voltage regulation using the approximate equivalent circuit.

2.20 A 1ϕ, 250 kVA, 11 kV/2.2 kV, 60 Hz transformer has the following parameters.

$$R_{\text{HV}} = 1.3\,\Omega \qquad X_{\text{HV}} = 4.5\,\Omega$$
$$R_{\text{LV}} = 0.05\,\Omega \qquad X_{\text{LV}} = 0.16$$
$$R_{\text{C(LV)}} = 2.4\,\text{k}\Omega \qquad X_{\text{m(LV)}} = 0.8\,\text{k}\Omega$$

(a) Draw the approximate equivalent circuit (i.e., magnetizing branch, with R_{c} and X_{m} connected to the supply terminals) referred to the HV side and show the parameter values.

(b) Determine the no-load current in amperes (HV side) as well as in per unit.

(c) If the low-voltage winding terminals are shorted, determine

(i) The supply voltage required to pass rated current through the shorted winding.

(ii) The losses in the transformer.

(d) The HV winding of the transformer is connected to the 11 kV supply and a load, $Z_{\text{L}} = 15\underline{/-90^\circ}\,\Omega$ is connected to the low-voltage winding.
Determine:

(i) Load voltage.

(ii) Voltage regulation $= \dfrac{|V_2|_{\text{load}} - |V_2|_{\text{no load}}}{|V_2|_{\text{load}}} \times 100.$

2.21 **(a)** The transformer is connected to a supply on the LV (low-voltage) side, and the HV (high-voltage) side is shorted. For rated current in the HV winding, determine:

(a) The current in the LV winding.

(b) The voltage applied to the transformer.

(c) The power loss in the transformer.

(b) The HV side of the transformer is now connected to a 2300 V supply and a load is connected to the LV side. The load is such that rated current flows through the transformer, and the supply power factor is unity. Determine:

(a) The load impedance.

(b) The load voltage.

(c) Voltage regulation (use Eq. 2.16).

2.22 A 1ϕ, 25 kVA, 2300/230 V transformer has the following parameters:

$$Z_{\mathrm{eq,H}} = 4.0 + j5.0\ \Omega$$
$$R_{\mathrm{c,L}} = 450\ \Omega$$
$$X_{\mathrm{m,L}} = 300\ \Omega$$

The transformer is connected to a load whose power factor varies. Determine the worst-case voltage regulation for full-load output.

2.23 For the transformer in Problem 2.22:

(a) Determine efficiency when the transformer delivers full load at rated voltage and 0.85 power factor lagging.

(b) Determine the percentage loading of the transformer at which the efficiency is a maximum and calculate this efficiency if the power factor is 0.85 and load voltage is 230 V.

2.24 A 1ϕ, 10 kVA, 2400/240 V, 60 Hz distribution transformer has the following characteristics:

$$\text{Core loss at full voltage} = 100\ \text{W}$$
$$\text{Copper loss at half load} = 60\ \text{W}$$

(a) Determine the efficiency of the transformer when it delivers full load at 0.8 power factor lagging.

(b) Determine the per-unit rating at which the transformer efficiency is a maximum. Determine this efficiency if the load power factor is 0.9.

(c) The transformer has the following load cycle:

No load for 6 hours

70% full load for 10 hours at 0.8 PF

90% full load for 8 hours at 0.9 PF

Determine the all-day efficiency of the transformer.

2.25 The transformer of Problem 2.24 is to be used as an autotransformer.

- **(a)** Show the connection that will result in maximum kVA rating.
- **(b)** Determine the voltage ratings of the high-voltage and low-voltage sides.
- **(c)** Determine the kVA rating of the autotransformer. Calculate for both high-voltage and low-voltage sides.

2.26 A 1ϕ, 10 kVA, 460/120 V, 60 Hz transformer has an efficiency of 96% when it delivers 9 kW at 0.9 power factor. This transformer is connected as an autotransformer to supply load to a 460 V circuit from a 580 V source.

- **(a)** Show the autotransformer connection.
- **(b)** Determine the maximum kVA the autotransformer can supply to the 460 V circuit.
- **(c)** Determine the efficiency of the autotransformer for full load at 0.9 power factor.

2.27 Reconnect the windings of a 1ϕ, 3 kVA, 240/120 V, 60 Hz transformer so that it can supply a load at 330 V from a 110 V supply.

- **(a)** Show the connection.
- **(b)** Determine the maximum kVA the reconnected transformer can deliver.

2.28 Three 1ϕ, 10 kVA, 460/120 V, 60 Hz transformers are connected to form a 3ϕ, 460/208 V transformer bank. The equivalent impedance of each transformer referred to the high-voltage side is $1.0 + j2.0\,\Omega$. The transformer delivers 20 kW at 0.8 power factor (leading).

- **(a)** Draw a schematic diagram showing the transformer connection.
- **(b)** Determine the transformer winding current.
- **(c)** Determine the primary voltage.
- **(d)** Determine the voltage regulation.

2.29 Three 1ϕ, 100 kVA, 2300/460 V, 60 Hz transformers are connected to form a 3ϕ, 2300/460 V transformer bank. The equivalent impedance of each transformer referred to its low-voltage side is $0.045 + j0.16\,\Omega$. The transformer is connected to a 3ϕ source through 3ϕ feeders, the impedance of each feeder being $0.5 + j1.5\,\Omega$. The transformer delivers full load at 460 V and 0.85 power factor lagging.

- **(a)** Draw a schematic diagram showing the transformer connection.
- **(b)** Determine the single-phase equivalent circuit.
- **(c)** Determine the sending end voltage of the 3ϕ source.
- **(d)** Determine the transformer winding currents.

2.30 Two identical 250 kVA, 230/460 V transformers are connected in open delta to supply a balanced 3ϕ load at 460 V and a power factor of 0.8 lagging. Determine

- **(a)** The maximum secondary line current without overloading the transformers.
- **(b)** The real power delivered by each transformer.
- **(c)** The primary line currents.
- **(d)** If a similar transformer is now added to complete the Δ, find the percentage increase in real power that can be supplied. Assume that the load voltage and power factor remain unchanged at 460 V and 0.8 lagging, respectively.

2.31 Three identical single-phase transformers, each of rating 20 kVA, 2300/230 V, 60 Hz, are connected Y–Y to form a 3ϕ transformer bank. The high-voltage side is connected to a 3ϕ, 4000 V, 60 Hz supply, and the secondary is left open. The neutral of the primary is not connected to the neutral of the supply. The voltage between the primary neutral and the supply neutral is measured to be 1200 V.

(a) Describe the voltage waveform between primary neutral and supply neutral. Neglect harmonics higher than third.

(b) Determine the ratio of (i) phase voltages of the two sides and (ii) line voltages of the two sides.

(c) Determine the ratio of the rms line-to-line voltage to the rms line-to-neutral voltage on each side.

2.32 A 1ϕ, 200 kVA, 2100/210 V, 60 Hz transformer has the following characteristics. The impedance of the high-voltage winding is $0.25 + j1.5\ \Omega$ with the low-voltage winding short-circuited. The admittance (i.e., inverse of impedance) of the low-voltage winding is $0.025 - j0.075$ mhos with the high-voltage winding open-circuited.

(a) Taking the transformer rating as base, determine the base values of power, voltage, current, and impedance for both the high-voltage and low-voltage sides of the transformer.

(b) Determine the per-unit value of the equivalent resistance and leakage reactance of the transformer.

(c) Determine the per-unit value of the excitation current at rated voltage.

(d) Determine the per-unit value of the total power loss in the transformer at full-load output condition.

2.33 A single-phase transformer has an equivalent leakage reactance of 0.04 per unit. The full-load copper loss is 0.015 per unit and the no-load power loss at rated voltage is 0.01 pu. The transformer supplies full-load power at rated voltage and 0.85 lagging power factor.

(a) Determine the efficiency of the transformer.

(b) Determine the voltage regulation.

2.34 A 1ϕ, 10 kVA, 7500/250 V, 60 Hz transformer has $Z_{eq} = 0.015 + j0.06$ pu, $R_c = 60$ pu, and $X_m = 20$ pu.

(a) Determine the equivalent circuit in ohmic values referred to the low-voltage side.

(b) The high-voltage winding is connected to a 7500 V supply, and a load of $5\underline{/90^\circ}$ is connected to the low-voltage side. Determine the load voltage and load current. Determine the voltage regulation.

2.35 A 1ϕ, 10 kVA, 2200/220 V, 60 Hz transformer has the following characteristics:

$$\text{No-load core loss} = 100\ \text{W}$$

$$\text{Full-load copper loss} = 215\ \text{W}$$

Write a computer program to study the variation of efficiency with output kVA load and load power factor. The program should

(a) Yield the results in a tabular form showing power factor, per-unit kVA load (i.e., X), and efficiency.

(b) Produce a plot of efficiency versus percent kVA load for power factors of 1.0, 0.8, 0.6, 0.4, and 0.2.